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Optimal Crop Choice, Irrigation Allocation, and the Impact of Contract Farming

Woonghee Tim Huh
Operations and Logistics Division, Sauder School of Businesss University of British Columbia, Vancouver, British Columbia V6T-1Z2, Canada, tim.huh@sauder.ubc.ca

Upmanu Lall
Columbia Water Center and The Department of Earth and Environmental Engineering, Columbia University, New York, New York 10027, USA, ula2@columbia.edu

The changing climate and concerns over food security are prompting a new look at the supply chain reliability of products derived from agriculture, and the potential role of contract farming as a mechanism to address climate and price risk while contributing toward crop diversification and water use efficiency is also emerging. In this study, the decision problem of a farmer associated with allocating his land among different crops with varying water requirements is considered, given that a subset of the crops may be associated with a forward contract that is being offered by a buyer. The problem includes a decision to acquire a certain amount of irrigation water capacity prior to the season and to allocate this capacity as irrigation water to be applied during the season to each of the crops selected. Rainfall in the growing season and the market price of each crop at the end of the season are considered to be random variables. Two stochastic programming models are developed to consider facets of this problem and to understand how contracts that reduce market price uncertainty from the problem may change the farmer's decision. The structural properties of these models are discussed, and selected implications are illustrated through an application to data from the Ganganagar district in Rajasthan, India.

Key words: crop planning; contract farming; irrigation; agriculture; India
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1. Introduction

1.1. Motivation

Food security is a key global social goal. According to a comprehensive report by the United Nations Food and Agriculture Organization (2002), which assesses the long-term development of world food and agriculture for the next three decades, the worldwide demand for agriculture products is projected to grow at a modest rate of 1.5% per year, and global shortages are unlikely to occur provided that appropriate interventions exist. However, serious supply shortages as well as land scarcity may persist at regional and local levels in conjunction with chronic or episodic (e.g., drought) water scarcity.

According to the United Nations Food and Agriculture Organization (2003), the land held for agricultural use during the past five decades has expanded by a mere 12% to 11% of the global land surface while arable land per capita has in fact declined by 40%. Such a decline has been compensated for by an increase in cropping intensities and crop yield growth. For example, the world average grain yield doubled to 2.4 tonnes per hectare from 1962 and 1996. This increase is expected to account for 80% of the future increases in crop production in developing countries. The ability to increase production yield is closely related to water, as the potential to improve yield is restricted in non-irrigated agriculture (which depends entirely upon rainfall and accounts for 60% of the production in developing countries). Irrigated agriculture, which covers only 20% of the total arable area, is responsible for 40% of the production. While irrigation is important for agricultural production, it accounts for a large portion (approximately 70%) of the global water usage taken from rivers and groundwater. For example, in India, agriculture constitutes at least 90% of water usage (Kumar et al. 2005). The importance of irrigation and its impact on water will continue to assume a greater role since the area equipped for irrigation is projected to increase by 20% by the year 2030, through an estimated investment of 25–50 billion US dollars per year.

Water is routinely cited as a critical resource limitation and a persistent crisis for many developing countries, particularly India and China (Briscoe 1999, Dinar and Subramanian 1997, Kinzelbach et al. 2004). As illustrated in Brown and Lall (2006), average annual rainfall, inter-annual variability in rainfall, and intra-annual variability in rainfall are highly correlated with the per capita national GDP (the exceptions being countries that have oil or other resources that stimulate their economy, e.g., Libya and Nigeria). The developing and poor countries are typically the ones with low to medium average annual rainfall, and high variability in that rainfall. This sensitivity to climate and water availability has the greatest impact in agriculture, which is the dominant water user.

Since a large proportion of the population in many countries (e.g., 70% in India) is rural and engaged in agriculture, crop failure assistance and rural livelihood support programs may constitute significant national expenses, even if they are relatively ineffective (Jha et al. 2007). Access to free (or nearly so) reliable water or energy sources to pump groundwater becomes a major political issue, and meeting this demand translates into significant subsidies and expenses for both water and energy, while impacting the other uses of these resources negatively (Monari 2002). To meet national food security goals in this highly variable climate, price supports for grains are offered and procurement targets are set. Since crop failure is financially protected and a firm price is offered in the event of crop success, farmers' crop planning decisions are determined by non-market forces. This, combined with the provision of free water, maintains suboptimal water usage strategies and limits the possible investment in efficient water infrastructure development and maintenance (Jha et al. 2007, Monari 2002).

The Green Revolution led to significant gains in crop yield through the application of fertilizer, irrigation, and improvements in the genetic strains of many crops. However, there has been little improvement in Asia in the last decade and very little change in Africa. The limited effectiveness of government in these programs is often cited. There is potential for a dramatic increase in crop yield per unit of water for existing crops through investments in appropriate irrigation methods and technology. However, most developing country farmers are caught in a poverty trap. They may not have the capital to invest in irrigation technology (according to the Food and Agriculture Organization 2003, costs can range from US$1000 to US$10,000 per hectare, not counting the development of a water storage or source) and may not have guaranteed access to the source of irrigation water. Consequently, when farmers are faced with uncertain rainfall, the ability to transition to a more effective technology is not there. If private capital is effectively invested, it could help stabilize the incomes of both farmers and corporations. This is the premise of contract farming, where corporations could engage in forward contracts with farmers and also supply needed inputs in exchange for a quality controlled product available in a timely manner. A more sophisticated approach could lead to dynamic crop choices that would consider the global marketplace and climate change as key variables (Bravo-Ureta and Pinheiro 1993, Hamdy et al. 2003). Greater private participation in crop selection and water supply could introduce local farmers to options that increase revenue, save water, and provide access to the global marketplace. Exploring how to achieve this transition to higher efficiency and productivity can provide an opportunity for sustainable agricultural development and water management. This transition requires an understanding of how farmers make decisions as well as what changes the farmers are able to adopt.

The primary focus of this study is to provide a simple and stylized model for the farmers' decision making process for crop planning and water management in the contract farming context. This model is used to develop insights into the interplay among crop selection and allocation, irrigation decisions, and contract farming. It is also hoped that our model will serve as a reference point for future empirical fieldwork as well as a prototype for building more detailed models.

1.2. Summary of Our Results

We consider the problem of a farmer who decides which crops to plant and how to allocate a fixed amount of land to the crops when a forward contract may or may not be present. These decisions are made while rainfall and crop prices are still uncertain. The farmer also decides the amount of irrigated water to be applied to each crop, which may occur before or after the rainfall amount becomes known. We formulate our problems as stochastic optimization problems, where uncertainties arise from rainfall and crop prices. We identify concavity properties, which not only enable us to understand the underlying properties of the farmer's problems and the relationship among several parameters, but also position our model of the farmer as a useful building block for more complicated models.

We also report the results of a computational investigation. Based on historical data in one of the districts in India, we consider the farmer's problem where the possible crop choices are rice and sugarcane. Generally, rice requires more water than sugarcane, but the nonlinearities in the yield rates of sugarcane and rice with respect to water lead to an interesting trade-off that depends on the water usage. Our results show that, as the expected price of sugarcane increases (e.g., due to the increase in the global demand for biofuels), not only does the overall net profit increase but more land is allocated to sugarcane; however, the irrigation capacity may increase or decrease depending on its cost. For a risk-averse farmer, as the degree of risk aversion decreases (which occurs as the farmer gains easier access to capital or as independent farmers merge to become a larger cooperative or corporation), the irrigation capacity decreases due to the farmer's reluctance to make an investment up front. In the case of contract farming, the farmer's decisions as well as his water usage are sensitive to the contract and cost parameters.

1.3. Literature Review and Our Contribution

There is a vast literature on the application of operations research and operations management to crop planning and other related problems in agriculture. Many of these studies are reviewed in Glen (1987) and Lowe and Preckel (2004), and the intention of this section is to highlight a limited number of related studies. A large portion of the literature is devoted to developing linear programming or other mathematical programming formulations for specific problems involving farm land allocation, crop rotation, harvesting, etc. (e.g., Audsley 1985, Cocks 1968, Heady 1954, Jolayemi and Olaomi 1978, Rae 1971). The primary focus of the aforementioned studies is to demonstrate the viability of mathematical programming tools in efficiently solving a range of proposed problems, whereas, in comparison, our objective also includes obtaining the structural characterization of the optimization problem.

Yield uncertainty is a common characteristic in agriculture, and several models have been developed to mitigate the risk of mismatch between supply and demand. Jones et al. (2001, 2002, 2003) study seed corn production and show the benefit of the second production opportunity. Kazaz (2004) and Allen and Schuster (2004) study the capacity and production quantity decisions under yield uncertainty for olive oil production and the Concord grape harvest, respectively. Recently, Kazaz and Webster (2011) examined how yield uncertainty affects the optimal decisions of a food processing company. In the manufacturing and production literature, the randomness in yield is typically represented as an exogenous random variable (e.g., Yano and Lee 1995), and the source of randomness is usually not investigated. In the agricultural setting, where water availability is the main source of yield uncertainty, it is reasonable to build a stochastic optimization model in which the recourse action depends on rainfall and irrigation. In the Maatman et al. (2002) study of the Central Plateau of Burkina Faso in West Africa, stochastic recourse decisions depend on the observed rainfall values, but there exists no control to affect the yield of crops. In contrast, we endogenize the stochastic yield rate by allowing the farmer to possibly increase water supply through irrigation (recourse decision). While there exist several papers that demonstrate the dependency of the crop yield rate on multiple factors or investigate the control problem of dynamically managing several factors to increase the yield rate, their results are specific to the specific site under investigation, and furthermore they do not consider decisions at a higher scope such as crop planning limitations imposed by resource availability (Drummond et al. 2004, Li et al. 2004, Liu et al. 2001, Shani et al. 2004). To our knowledge, we are unable to identify any existing model for optimal crop planning that explicitly incorporates irrigation decisions under rainfall uncertainty.

The model in this study is general enough to study contract farming in a single framework. Under contract farming, the farmer enters into a contract with a buyer to sell the harvest before the beginning of the selling season. While there are several types of contracts used in contract farming, one of the most popular contracts is the *forward contract*, which specifies both the quantity and price in the future transaction. Under contract farming, the buyer can potentially benefit from the reduced risk of purchase price and supply availability, and the farmer can also benefit from access to financial resources and technology (Rehber 1998). Yet, it is not clear whether and how the current practice of contract farming benefits all the parties involved. Existing research on contract farming is mostly empirical or case-based (e.g., Runsten and Key 1996, Warning and Key 2002). There are a limited number of recent analytic models for contracts in the agriculture business (e.g., Burer et al. 2008, He and Zhang 2008), but these models do not address the contract farming problem directly.

A well-known concept in operations management similar to contract farming is "contract manufacturing." The motivating rationale for such an arrangement is core competency, where the original equipment manufacturer (buyer) focuses on product innovation and the contract manufacturer (supplier) focuses on capacity utilization through pooling (Plambeck and Taylor 2005, Ülkü et al. 2007). Contract farming differs from contract manufacturing in several aspects. The primary purpose of the buyer in contract farming is to ensure quantity and price stability of the supply instead of mitigating the risk associated with investment. Whereas the contract manufacturer achieves the pooling effect by supplying to multiple buyers, the contract farmer supplies to only one buyer, who procures from a large number of farmers in each season. The random yield effect has a more dramatic effect in the agricultural setting, and the implementation of contract farming can potentially infringe upon more public policy issues than its manufacturing counterpart.

In this study, we propose a stochastic programming model for studying a farmer's crop planning problem that explicitly accounts for rainfall uncertainty and irrigation decisions. Our model enables us to understand the relationship among several parameters of the model and their impact on one of the most important but often neglected resources, namely, water. It also provides a unified and parsimonious framework for studying the impact of risk aversion and contract farming. Our study can be viewed as one of the steps toward extending the operations literature to the globalization of agriculture, following the call of research by Lowe and Preckel (2004). As Schweigman et al. (1990) noted, agriculture is one of the areas where operations research can make significant contributions in developing countries. From a technical perspective, the concavity properties that we identify for our formulations do not follow from standard convexity preservation results, and we overcome this difficulty by using an approach based on the first principles.

2. Basic Model: With Static Irrigation Control (Model S)

We assume that the crop allocation and irrigation capacity decisions are made at the beginning, before any of the uncertainties are resolved, and market prices for crops are realized at the end after all the decisions are made. For the modeling of the irrigation usage decision (the quantity and allocation of irrigated water) and rainfall realization, the most realistic modeling approach would be a multi-period dynamic control of irrigated water that depends on the evolution of rainfall during the season; however, such an approach would be computationally intensive and may not produce additional insights, and thus we introduce simplifications. In this section, we introduce the case where the irrigation usage decision precedes rainfall realization, which we refer to as the *Model with Static Irrigation Control*, or simply Model **S**. The other case where this decision follows rainfall realization is referred to as the *Model with Responsive Irrigation Control* or Model **R**, and it is discussed in section 4.

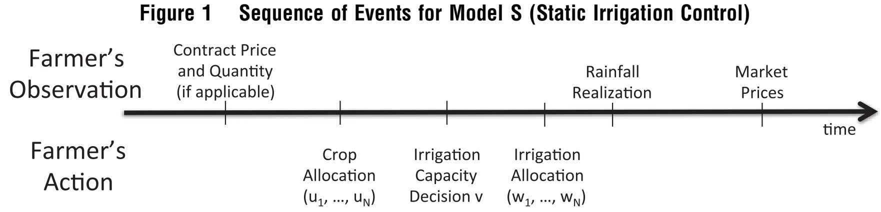
2.1. Description

We consider the decision of the farmer, who has control over a fixed area of agricultural land, and this area cannot be increased or decreased during the planning horizon. Without loss of generality, the total area of land is 1 unit. We assume that the parameters of a contract, if it exists, are already determined, and they are exogenous to our model and known to the farmer. There are N types of potential crops, indexed by $i = 1,\ldots,N$. We assume that the following sequence of events occurs—see Figure 1 for an illustration. (Below, "A" represents *action* and "O" represents *observation*.)

(A) At the beginning of the year, the farmer plants a subset of potential crops for the upcoming farming season. Let u_i represent the amount land allocated to crop i. Thus, $u_1 + \cdots + u_N = 1$. In addition, he may decide to invest in the irrigation project, which provides additional water for his farmland during the season. Let v be the irrigation capacity, which is the maximum amount of water available for irrigation, and this capacity v is also a part of the farmer's decisions.[1] (If the farmer does not invest in the irrigation project, then $v = 0$.) Furthermore, the farmer also makes the decisions regarding how the irrigated water will be used. His allocation should satisfy the capacity constraint that the total amount of irrigated water cannot exceed v. Let $w_i \geq 0$ denote the total amount of water irrigated to crop i. Then, $w_1 + \cdots + w_N \leq v$. If the farmer distributes w_i amount of water unevenly in the farmland of crop i, it can be shown to be suboptimal, and thus we assume without loss of generality an even distribution of water within each crop area.

(O) Let $R \geq 0$ denote the total amount of rainfall in a season, which we assume to fall evenly in the land. The value of the exogenous random variable R is realized, and we denote the realized value by r. (While the farmer observes the realized rainfall r, we assume that he cannot block rainfall from a portion of his land or divert it to another portion.) The yield amount of each crop is given by the amount of land allocated to the crop, u_i, multiplied by the yield rate. The yield rate of each crop, γ_i, depends on the total amount of water input, which is the sum of the rainfall and water

Figure 1 Sequence of Events for Model S (Static Irrigation Control)



irrigation. The actual market prices for crops are also realized, and the actual payoff to the farmer is based on these prices as well as the contract parameters, if applicable.

We now describe the components of the farmer's payoff function (objective function). Our model is flexible enough to accommodate contract farming and the farmer's risk aversion.

Crop Yield Rate. We use $\gamma_i(y)$ to denote the yield rate of crop i, which represents the quantity of crop i harvested from a unit area of farmland, given that the total amount of water input is y. The total amount of water y is the sum of (i) the rainfall and (ii) the irrigated water. We model the yield rate as a function of the total amount of input water only. (In the case of wheat, it has been identified that the key drivers for the yield rate are water and fertilizers usages (Li et al. 2004). We exclude the fertilizer decision in our model, which is a reasonable modeling assumption *if* the optimal amount of fertilizer can be dispatched relatively easily since its availability is less constrained by stochastic elements of the system (e.g., rainfall), or *if* fertilizer is not available at all. The dominant effect of water on the yield has also been recognized by the empirical model of Kumar and Khepar (1980), who have developed the relationship between water input and production yield in India, and also by the operations research model of Schweigman et al. (1990), who have employed statistical regression for studying the rainfall data in Tanzania.

As the amount of water input increases, it is reasonable to expect that the yield rate increases initially; however, an excessive amount of water is not desirable. We model that γ_i is a concave function with a finite maximum (e.g., Kumar and Khepar 1980). For simplicity of exposition, we assume that the maximizer of γ_i is unique and positive, and γ_i is strictly increasing between 0 and the maximizer. (In reality, the yield rate will be zero unless the total water input exceeds a certain minimum threshold quantity, in which case, the yield rate function fails to be concave. However, unless there is an extreme drought or flood, it is reasonable to expect that this function for the crop of our interest is concave within a reasonable domain of water input.)

Suppose that the amount of irrigated water, w_i, is distributed in the area of u_i. Then, the amount of irrigated water normalized for per unit area is $z_i = w_i/u_i$. Since r represents the amount of rainfall per unit area, the total amount of water a unit area would receive is $r + w_i/u_i$, and the yield rate of this area is a function of this quantity:

$$\gamma_i(r + w_i/u_i).$$

The realized amount of crop i is given by the amount of land allocated to crop i multiplied by the yield rate, which is

$$x_i = u_i \cdot \gamma_i(r + w_i/u_i) = u_i \cdot \gamma_i(r + z_i).$$

Irrigation Cost. We model the cost of irrigation with the following two components: capacity investment and usage. The capacity investment cost is the upfront cost spent to ensure the availability of irrigated water. It may include, for example, drilling a well or building a water reservoir or acquiring water rights (where the cost is annualized). We assume that the investment cost is proportional to the irrigation capacity v. The usage cost is associated with the actual amount of water irrigated to the land and includes the cost of electricity for pumping water from a well or the transportation cost for water from the reservoir to the farmland. We also assume that the usage cost is linear in the amount of irrigated water. Thus, the total cost of irrigation, $\mathcal{C}$, is given by the sum of $\kappa^1 \cdot v$ and $\kappa^2 \cdot (w_1 + \cdots + w_N)$, where $\kappa^1, \kappa^2 \geq 0$. Since $w_i = u_i \cdot z_i$, it follows that this cost can be written as

$$\mathcal{C} = \mathcal{C}(v, z_1, \ldots, z_N) = \kappa^1 \cdot v + \kappa^2 \cdot \sum_{i=1}^{N} u_i \cdot z_i.$$

(Note that $\mathcal{C}$ depends on $(u_1, \ldots, u_N)$, but we suppress such dependence in our notation for brevity.)

Net Revenue and Contract Farming. Let $\mathbf{x} = (x_1, \ldots, x_N)^T$ denote the total yield vector, where $x_i = u_i \cdot \gamma_i(r + z_i)$ and $z_i = w_i/u_i$. Let $\mathbf{P} = (P_1, \ldots, P_N)^T$ denote the vector of nonnegative random variables, each corresponding to the market price of crops. These random variables, $P_1, \ldots, P_N$, may be correlated. Then, the farmer's revenue without any contract is given by

$$\mathbf{P} \cdot \mathbf{x} = \sum_{i=1}^{N} P_i \cdot x_i.$$

Note that the farmer's revenue is separable in $\mathbf{z}$. Furthermore, it is also linear in this case.

Now, we show how we can incorporate contract farming into our model. We consider the commonly used forward buy/sell contract, where the farmer and the buyer commit to both the quantity and the price at the beginning of the planning horizon. Let $\{(p_i^F, q_i^F) : i = 1, \ldots, N\}$ represent the family of forward contracts. (In our model, we allow the possibility of multiple contracts, one for each crop, though in practice a farmer typically commits to at most one contract. If $q_i^F = 0$, it indicates the absence of a forward contract for crop i.) The specification of these contracts is exogenous to our model, and consequently neither the contract price nor the quantity are considered explicitly as decision variables. This situation can be changed where we consider a more sophisticated model of the interaction between the farmer and the contract offerer. For now, we investigate the effects of changes in these variables parametrically. If the farmer produces exactly the quantity specified by the contract, then he earns $p_i^F \cdot q_i^F$ from crop i. If the farmer produces more than the forward contract amount, then he sells any excess amount of the crop to the market at the spot price; thus, the additional revenue from the market for crop i is $P_i \cdot [x_i - q_i^F]^+$ where x_i is the production quantity of crop i. Otherwise, if the farmer does not produce enough quantity to satisfy the forward contract, we assume that he must pay the buyer a penalty cost proportional to the shortage amount. (The shortage penalty possibly represents additional costs such as emergency activities and transportation and acquisition of supply by the contract offerer at spot market prices.) Let $\mathbf{b} = (b_1, \ldots, b_N)$ be the vector of per-unit shortage penalty costs. Furthermore, we allow $\mathbf{b}$ to depend on $\mathbf{P}$ and require only that the penalty cost satisfies $\mathbf{b} \geq \mathbf{P}$ (for example, $b_i = P_i + \zeta_i$ where $\zeta_i \geq 0$ or $b_i = \zeta_i \cdot P_i$ where $\zeta_i \geq 1$). This constraint of $\mathbf{b}$ based on $\mathbf{P}$ rather than on $\mathbf{p}^F$ addresses the situation where the farmer may default on the contract if the market price were higher than the contract price. Then, the farmer's net revenue is given by

$$\mathcal{R} = \mathcal{R}(\mathbf{x}) = \mathbf{p}^F \cdot \mathbf{q}^F + \mathbf{P} \cdot \left[\mathbf{x} - \mathbf{q}^F\right]^+ - \mathbf{b} \cdot \left[\mathbf{q}^F - \mathbf{x}\right]^+,$$

where $\mathbf{q}^F = (q_1^F, \ldots, q_N^F)$ and $\mathbf{p}^F = (p_1^F, \ldots, p_N^F)$.

The following result shows that the net revenue function is a well-behaved function of $\mathbf{x}$, even with the presence of the contract.

Proposition 1. *For any $\mathbf{P}$, $\mathcal{R}(\mathbf{x})$ is separable, nondecreasing and concave in $\mathbf{x}$.*

Risk Aversion and Payoff Function. Recall that $\mathcal{C}$ denotes the irrigation cost and $\mathcal{R}$ denotes the net revenue from crop yields. We then define the farmer's profit by computing $\mathcal{R}$ minus $\mathcal{C}$. To incorporate the farmer's aversion to the stochasticity in revenue, we take the approach of evaluating the expected utility of profit, where the utility function is a single-dimensional function $\mathcal{U} : \Re \rightarrow \Re$ that is both increasing and concave. The farmer's objective is to maximize his expected utility, which we refer to as his payoff:

$$\max \quad \Pi = E[\mathcal{U}(\mathcal{R} - \mathcal{C})],$$

where the expectation is taken over rainfall R and crop prices $\mathbf{P}$. Note that Π is a function of $\mathbf{u} = (u_1, \ldots, u_N)$, v and $\mathbf{w} = (w_1, \ldots, w_N)$. If $\mathcal{U}$ is an identity function, then Π is simply $E[\mathcal{R} - \mathcal{C}]$, and we refer to this special case as the *risk neutral* case.

2.2. Stochastic Programming Formulation

We now present a mathematical programming formulation of the stochastic optimization problem. In the description of section 2.1, the farmer's decisions are the triplet of $\mathbf{u}$, v, and $\mathbf{w}$. It is convenient to replace $\mathbf{w} = (w_1, \ldots, w_N)$ with $\mathbf{z} = (z_1, \ldots, z_N)$ using the relationship $z_i = w_i / u_i$. None of these decisions depends on the realizations of R or $\mathbf{P}$. The formulation is given below:

$$\text{(SP-S)} \quad \max_{\mathbf{u},v,\mathbf{z}} E_{R,\mathbf{P}}\left[\mathcal{U}(\mathcal{R}(u_1 \cdot \gamma_1(z_1 + R), \ldots, u_N \cdot \gamma_N(z_N + R)) - \mathcal{C}(v, z_1, \ldots, z_N))\right]$$
$$\text{s. t.} \quad \sum_{i=1}^{N} u_i \cdot z_i \leq v \quad u_1 + u_2 + \cdots + u_N = 1$$
$$v \geq 0, u_i \geq 0, z_i \geq 0 \quad \text{for each } i = 1, \ldots, N.$$

2.3. Analysis

We now describe the properties of (SP-S) that are useful in developing an algorithm for finding an optimal solution for this formulation. Since both the objective function and the feasible region of (SP-S) fail to be convex, such problems are in general analytically difficult to solve. However, we can show that our problem can be written as a sequence of convex programming formulations, and consequently it becomes easy to solve.

Water Distribution Within a Single Crop. We first comment on how irrigated water should be distributed within the given crop area. If w_i amount of irrigated water is available for crop i, should it be distributed evenly or concentrated in a smaller portion of the crop area? It can be argued that it is optimal to distribute w_i amount of irrigated water evenly throughout the crop area of size u_i. This retroactively justifies the even distribution assumption that we had already introduced.

Optimal Irrigation Capacity and Usage Decision. While the decision variables in (SP-S) consist of ($\mathbf{u}$,v, $\mathbf{z}$), we restrict our attention to a smaller problem of optimizing over (v,$\mathbf{z}$) while holding $\mathbf{u}$ fixed. Suppose $u_1 + u_2 + \cdots + u_N = 1$. Note that in any optimal solution, $v = w_1 + \cdots + w_N = u_1 z_1 + \cdots + u_N z_N$ must always be satisfied (since both irrigation capacity and allocation decisions are made at the same time, and there is no reason to reserve irrigation capacity that is not used in Model **S**). Let $\Pi(\mathbf{u},v,\mathbf{z})$ denote the objective function of (SP-S). Define

$$\text{(SP-Sa)} \quad \phi(\mathbf{u}) = \max_{\mathbf{z}} \Pi\left(\mathbf{u}, \sum_{i=1}^{N} u_i z_i, \mathbf{z}\right)$$
$$\text{s. t. } z_i \geq 0 \quad \text{for each } i = 1, \ldots, N,$$

which is an optimization problem with respect to $\mathbf{z}$ for fixed $\mathbf{u}$. The following results follow from standard results of convex analysis.

THEOREM 2. *(SP-Sa) is a concave function maximization problem with nonnegativity constraints with respect to* $\mathbf{z} = (z_1, \ldots, z_N)$.

There exist many efficient algorithms to solve concave function maximization problems with linear constraints. If the farmer is risk neutral, then we can further characterize the optimal solution. Since the net revenue function $\mathcal{R}$ is separable, it can be written as $\mathcal{R}(x_1, \ldots, x_N) = \mathcal{R}_1(x_1) + \cdots + \mathcal{R}_N(x_N)$ for certain single-variable functions $\{\mathcal{R}_1, \ldots, \mathcal{R}_N\}$. By the concavity and monotonicity of $\mathcal{R}$, each $\mathcal{R}_i$ is nondecreasing and concave. Thus, $E_{R,P_i}[\mathcal{R}_i(u_i \cdot \gamma_i(z_i + R))]$ is also concave in z_i. The following result shows that the optimal solution to (SP-Sa) can be found by optimizing over each component separately.

PROPOSITION 3. *In the risk neural case, an optimal solution* $\mathbf{z}$ *to (SP-Sa) is given by, for each* $i = 1,\ldots,N$,

$$\frac{\partial}{\partial z_i} E_{R,P_i}[\mathcal{R}_i(u_i \cdot \gamma_i(z_i + R))] = (\kappa^1 + \kappa^2) \cdot u_i.$$

OPTIMAL CROP ALLOCATION DECISION. In our discussion of (SP-Sa), we have discussed the problem of finding the optimal decisions of $\mathbf{z}$ for given $\mathbf{u}$, where $\phi(\mathbf{u})$ denotes the optimal value of (SP-Sa). We now consider the problem of optimizing ϕ over $\mathbf{u}$, i.e.,

$$\text{(SP-Sb)} \quad \max_{\mathbf{u}} \phi(\mathbf{u}) \text{ s. t.} \sum_{i=1}^{N} u_i = 1$$
$$u_i \geq 0 \quad \text{for each } i = 1, \ldots, N.$$

Our objective is to show the concavity of φ, which is useful in finding the optimal solution for (SP-Sb). In fact, we will show the convexity result for every sample path of r. We note that this result does *not* follow from the standard convexity result of convex programs (e.g., Rockafellar 1970, Chapter 29, Boyd and Vandenberghe 2004, Chapter 5) since the objective function contains a non-convex term $u_i \cdot \gamma_i(z_i + R)$. The main idea in the proof of Theorem 4 is to carefully construct a $\mathbf{z}$ vector for a midpoint of a pair of arbitrary choices of $\mathbf{u}$ vectors, such that the constructed $\mathbf{z}$ vector is feasible and yields sufficiently high objective value.

THEOREM 4. *The optimal value of (SP-Sb) is concave in the first stage decisions* $(u_1, \ldots, u_N)$.

Thus, in this section, we have shown that while (SP-S) for Model **S** is not a convex problem, it can be written as a sequence of convex problems that can be solved easily.

3. Numerical Results

In this section, we present numerical results based on empirical data. In section 3.1, we present data and our model parameters. In sections 3.2 and 3.3, we analyze the risk neutral and risk averse cases, and then study the impact of contract farming in section 3.4. The discussion in this section is based on Model **S** of section 2.

3.1. Data

The numerical computation of this study is based on data from the Ganganagar district, in the state of Rajasthan in northwestern India, where the majority of the population depends on agriculture. This region has witnessed a transformation of the landscape in the last century as the Gang canal has brought excess water from Punjab and Himachal Pradesh to a region with comparatively low annual rainfall (around 300 mm). We focus on this region because of the relative importance of irrigation in this region and also because the Rajasthan state government has encouraged contract farming. Wheat and cotton are the major crops grown in this district, but significant amounts of rice and sugarcane are also grown.

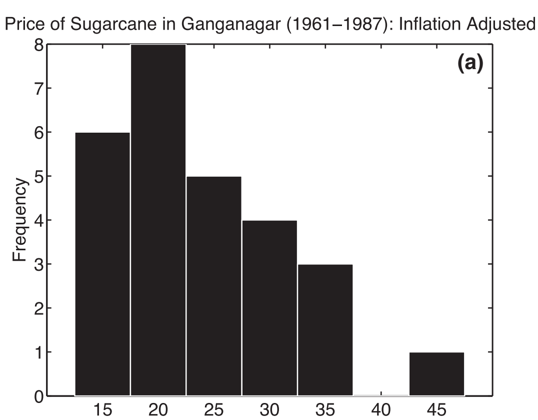
We take historical prices for rice between years 1961 and 1987 from an agricultural data set from the World Bank (Sanghi et al. 1998). (This database contains various statistics pertaining to Indian agricultural, climatological, edaphic and geographical variables.) The data set does not include the sugarcane prices but includes the sugar prices, and we take scalar multiples of sugarcane prices to be sugar prices. (The multiplier is chosen under the assumptions that 100 kg of sugarcane is required to produce 8.5 kg of sugar and that 60% of the cost of sugar is due to the cost of sugarcane. These assumptions are based on and consistent with various sources such as India Infoline 2002, Gill 2000, The Hindu 2006, Multi Commodity Exchange of India Ltd 2006, and Sri Lanka Sugarcane Research Institute 2005.) The prices are adjusted to the 1987 level using the annual Indian inflation rates for agricultural commodities, available from the *World Development Indicators 2007*, which is the World Bank's annual compilation of data concerning development. The annual rainfall data in this region is based on the historical data between the years 1951 and 2002 (52 data points), available from the Indian Meteorological Department (2007). The empirical data of the inflation-adjusted prices and rainfall have the average values of

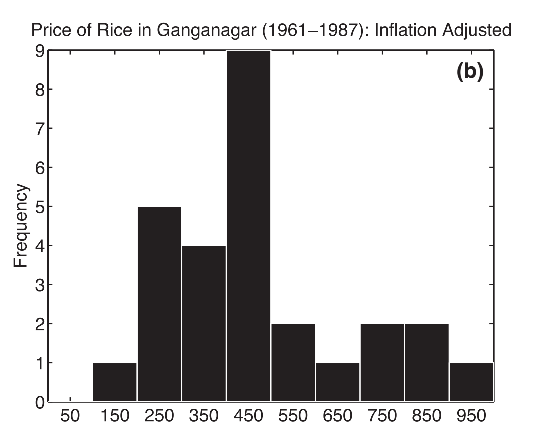
$$E[P_{\text{rice}}] = 470.1, E[P_{\text{sugarcane}}] = 24.09, \text{ and } E[R] = 263.2,$$

and their histograms are shown in Figures 2 and 3. Furthermore, we observe that both the prices and rainfall are significantly variable, as indicated by following standard deviations: $\sigma[P_{\text{rice}}] = 206.9$, $\sigma[P_{\text{sugarcane}}] = 8.06$, and $\sigma[R] = 94.71$. In addition, data show that the prices of two crops are positively correlated whereas price and rainfall are negatively correlated; more precisely, the correlation coefficients are $\rho[P_{\text{rice}}, P_{\text{sugarcane}}] = 0.44$, $\rho[P_{\text{rice}}, R] = -0.20$, and $\rho[P_{\text{sugarcane}}, R] = -0.31$.

For our computation, since the random variables are correlated, we use the *k*-nearest neighbor bootstrap method (Lall and Sharma 1996) to generate samples of $(P_{\text{rice}}, P_{\text{sugarcane}}, R)$. For R, we use 27 years of data between 1961 and 1987 (for which the corresponding price data are available). Then, for each value of R, we choose samples of $(P_{\text{rice}}, P_{\text{sugarcane}})$ that correspond to similar values of R. More precisely, let $\{(p^t_{\text{rice}}, p^t_{\text{sugarcane}}, r^t) : t = 1, \ldots, 27\}$ denote the historical data. Let (t) denote the sorted index for r^t's, that is, $r^{(1)} \leq \ldots \leq r^{(27)}$. We sample R uniformly from $\{r^{(1)}, \ldots, r^{(27)}\}$. For any $r^{(t)}$, we choose from $(p^{(t-2)}, p^{(t-1)}, p^{(t)}, p^{(t+1)}, p^{(t+2)})$ with probability proportional to $(\frac{1}{3}, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{3})$. (We make necessary truncations if t is either too small or too large.)

Figure 2 The Histograms of the Inflation-Adjusted Price Distributions (rupees/qt): 1961–1987. Based on 27 Observations in Total for Each





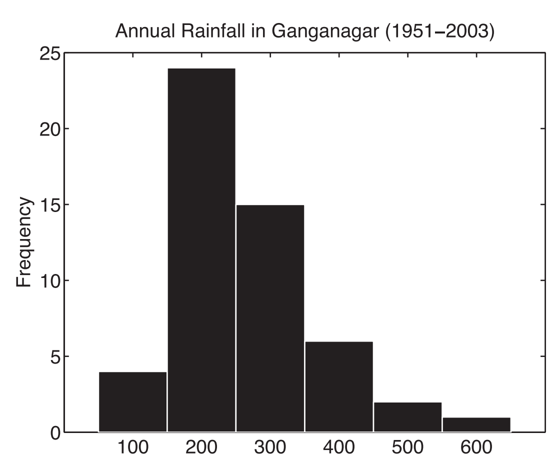
We model the yield rates as a function of water input using the water-production model estimated by Kumar and Khepar (1980); in particular,

$$\gamma_{\text{rice}}(y) = 5.9384 - 0.035206y + 2.412043y^{0.5}$$
$$\gamma_{\text{sugarcane}}(y) = -11.5441 + 2.92837y - 0.0027y^2,$$

where the input variable y is the total amount of water in millimeters per hectare, and the output $\gamma(y)$ is given in quintal per hectare. (One quintal is equivalent to 100 kg or 0.1 t.) Each of these functions is concave. For rice, γ_{rice} is maximized at 1173.48 mm/ha achieving the output value of 47.25 qt/ha, and for sugarcane, $\gamma_{\text{sugarcane}}$ is maximized at 542.30 mm/ha achieving the value of 782.50 qt/ha (see Figure 4). Note that, in general, rice requires a greater amount of water than sugarcane does, but the yield rate of sugarcane is more sensitive to water input than that of rice. Furthermore, the amount of water input available from rainfall is insufficient for the maximum yield of both crops.

For the irrigation cost, we vary the irrigation capacity investment parameter κ^1 in the range of 2–20 rupees per millimeter-hectare. (This range was derived based on the Food and Agriculture Organization 2003 estimate of the infrastructure cost of US\$1000 to US\$10,000/ha, using the annualized investment cost rate of 15% under the assumption that the estimate was based on the capacity of 1000 mm. Inflation and exchange rate data are taken from International Monetary Fund and Yahoo finance.) For

Figure 3 The Histogram of the Rainfall Distributions (mm): 1951–2002. Based on 52 Observations in Total



simplicity, we fix $\kappa^2 = 0$, that is, irrigation usage is costless.

For the utility function, we use the following increasing concave function for the utility function: $\mathcal{U}(z) = [1 - \exp(-\lambda z)]/\lambda$ where $\lambda > 0$. Since $\mathcal{U}(z) \to z$ as $\lambda \downarrow 0$, we also define $\mathcal{U}(z) = z$ if $\lambda = 0$. We note that this utility function displays the "constant absolute risk aversion" property, that is, $-\mathcal{U}''(z)/\mathcal{U}'(z)$ is a constant.

3.2. Without Contracts: Risk Neutral Case

We first consider the payoff function for the risk-neutral farmer in the absence of any forward contract. Here, the expected revenue is given by $\mathcal{R}(\mathbf{x}) = E[P_{\text{rice}}] \cdot x_{\text{rice}} + E[P_{\text{sugarcane}}] \cdot x_{\text{sugarcane}}$ where x_{rice} and $x_{\text{sugarcane}}$ represent the production quantities of rice and sugarcane, respectively. In this section, we fix the price distribution of rice, but multiply the distribution of sugarcane by various scalars to study the effect of price changes in sugarcane. We also vary the cost parameter κ^1 for irrigation investment.

The computational results are summarized in Figure 5. In all three figures, the horizonal axis represents a range of values of κ^1, and each line is indexed by a multiplicative adjustment factor for the price of sugarcane, $P_{\text{sugarcane}}$, to study the impact of changes in the relative price. (Thus, the adjustment factor of 1.0 represents no modification.) In Figure 5a, we observe that the payoff objective function expectedly decreases in the cost parameters κ^1 and increases in the expected selling price of sugarcane, $E[P_{\text{sugarcane}}]$.

Figure 5b shows the allocation of land between the two crops. Without any price adjustment, the crop allocation is always to plant rice (i.e., $u_{\text{rice}} = 1$), for all

Figure 4 Yield Rates (qt/ha) as a Function of Water Input (mm/ha)

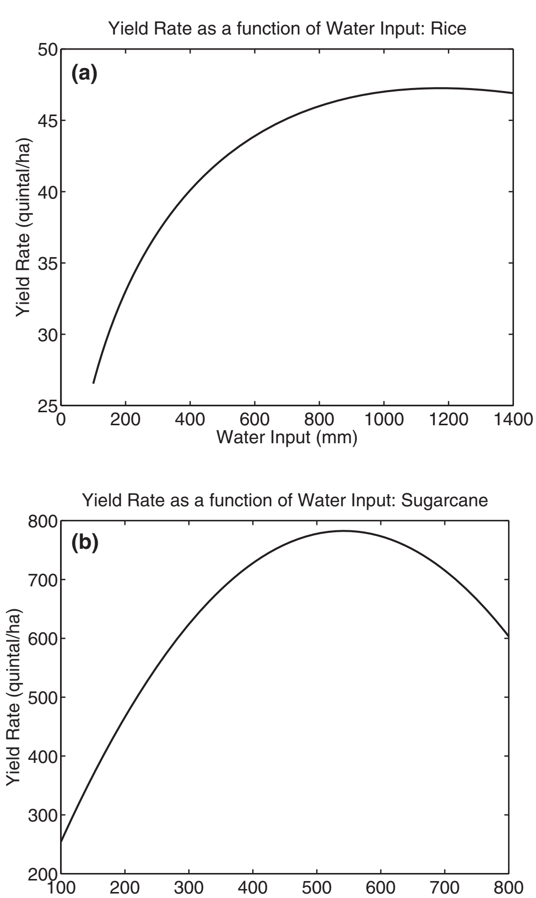
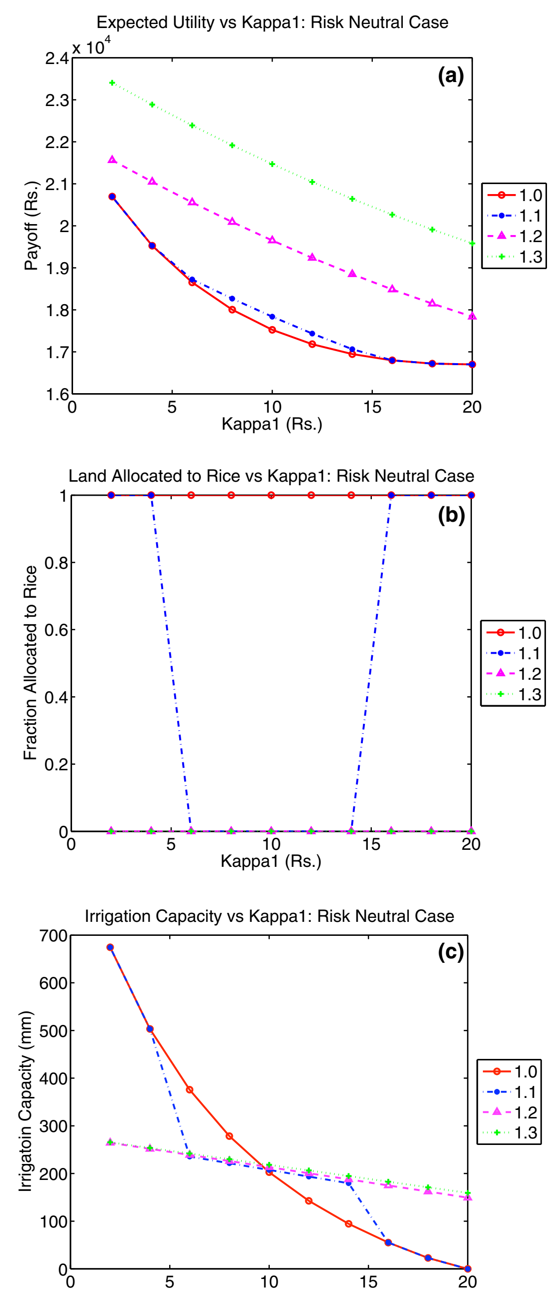


Figure 5 Without Contract: Risk Neutral Case (Model S). Legend Represents the Multiplicative Adjustment for the Price of Sugarcane $P_{\text{sugarcane}}$. (a) Payoff Function Π vs. κ^1. (b) Crop-Land Allocated to Rice u_{rice} vs. κ^1. (c) Irrigation Capacity v vs. κ^1.



values of κ^1 tested. However, when the price of sugarcane is 10% higher, as κ^1 increases, the crop allocation switches from rice to sugarcane since sugarcane requires less water than rice (γ_{rice} is maximized at 1173.48 mm/ha whereas $\gamma_{\text{sugarcane}}$ is maximized at 542.30 mm/ha). The allocation, however, eventually switches back to rice, which is less sensitive to water input than sugarcane. When the price of sugarcane is much higher (20% or higher), it is more profitable to plant sugarcane only.

We consider the effect on the water resource in Figure 5c, which shows an unsurprising result that more water is used when the cost of irrigation is affordable. The comparison of all rice and all sugarcane (corresponding to the adjustment factors of 1.0 and 1.2, respectively) shows that the sensitivity of the irrigation capacity on its cost parameter κ^1 is higher in rice than in sugarcane. This observation can be explained by the fact that the yield curve of sugarcane is comparatively more sensitive, implying that bringing the total water input to its maximizer becomes crucial. A few implications of our findings are (i) the crop-by-crop land allocation, with disregard to the irrigation cost, is not necessarily a good indicator for estimating water usage and (ii) reducing the farmer's cost of irrigating water (e.g., through subsidy) has quite a substantial impact on the water usage.

3.3. Without Contracts: Risk Aversion

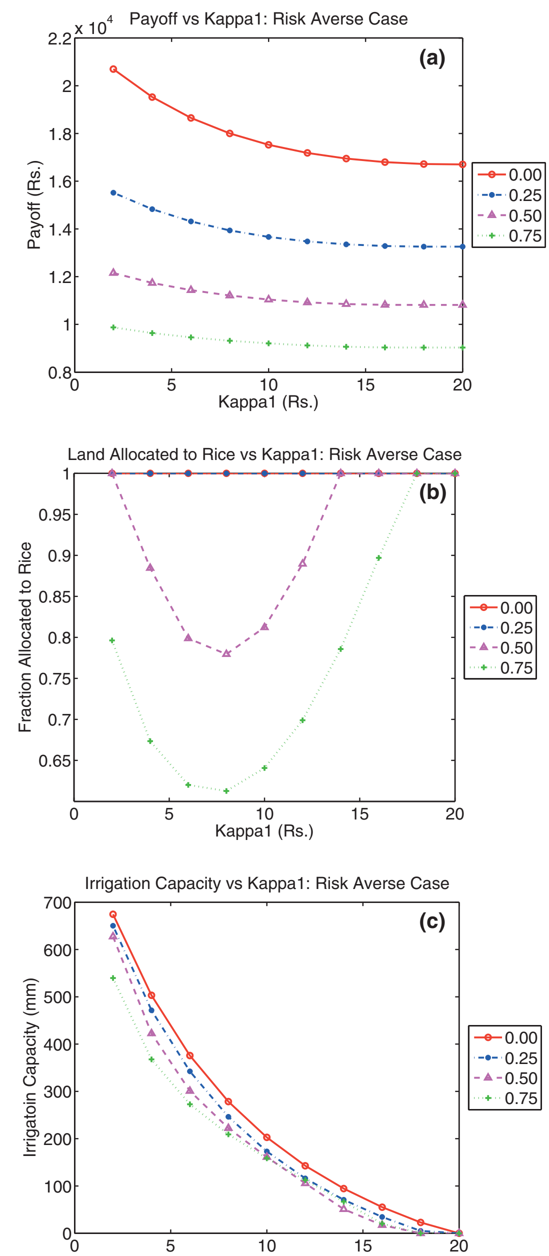
To study the effect of risk aversion, we test various values of the parameter λ, a proxy for the degree of risk aversity. From the definition of the utility function $\mathcal{U}$, a higher value of λ implies lower payoff, which is shown in Figure 6a. This figure also shows that the expected payoff decreases in the irrigation cost parameter κ^1, as expected.

Figure 6b shows that while the risk-neutral farmer ($\lambda = 0$) plants rice only, some mix of sugarcane becomes more preferable as the farmer becomes more risk averse ($\lambda = 0.50 \times 10^{-4}$ or 0.75×10^{-4}). This observation can be explained by the fact that diversification is generally a good strategy against risk (as in financial assets). Moreover, we note that the farmer can easily control the yield rate of sugarcane by satisfying its modest water requirement; for rice, however, it becomes too expensive to prepare enough irrigation for the maximum yield of rice and thus its yield rate is subject to the outcome of the rainfall. This explains why the rice is the more risky crop, which the risk averse farmer likes to avoid.

In Figure 6c, we see that irrigation capacity tends to decrease in the risk aversion parameter λ as well as the irrigation cost κ^1, as the farmer becomes reluctant to spend an upfront investment that may or may not become useful. It has an important policy implication since the initiatives to reduce the irrigation cost to the farmer (e.g., through government subsidy) and to decrease the farmer's risk aversion (e.g., through crop insurance, corporatization of farming and easy access to capital), albeit politically appealing, are likely to result in the higher usage of irrigation if appropriate interventions fail to exist.

In summary, we see that, in the absence of any contract, risk aversion of the farmer leads to crop diversification. It also discourages the farmer from investing in irrigation capacity since the risk-averse farmer

Figure 6 Without Contract: Risk Averse Case (Model S). Legend Represents the Risk Aversion Parameter γ in 10^{-4} (a) Payoff Function Π vs. κ^1. (b) Crop-Land Allocated to Rice u_{rice} vs. κ^1. (c) Irrigation Capacity v vs. κ^1.



would like to avoid the up-front cost; thus, measures to reduce the risk aversion of the farmer may increase overall water usage.

3.4. With Forward Contracts

We now consider the impact of contract farming by incorporating a forward contract into the computational analysis. We assume that there is no contract for rice and that the forward contract for sugarcane is given by two parameters $q^F_{\text{sugarcane}}$ and $p^F_{\text{sugarcane}}$. We fix the penalty cost at 150% of the market price, that is, $b_{\text{sugarcane}} = 1.5 \cdot P_{\text{sugarcane}}$. If x_{rice} and $x_{\text{sugarcane}}$ represent the total yield of rice and sugarcane, respectively, then the profit function is given by

$$\begin{aligned}\mathcal{R} - \mathcal{C} = {} & P_{\text{rice}} \cdot x_{\text{rice}} + p^F_{\text{sugarcane}} \cdot q^F_{\text{sugarcane}} \\ & + P_{\text{sugarcane}} \cdot [x_{\text{sugarcane}} - q^F_{\text{sugarcane}}]^+ \\ & - b_{\text{sugarcane}} \cdot [q^F_{\text{sugarcane}} - x_{\text{sugarcane}}]^+.\end{aligned}$$

In the definition of the utility function $\mathcal{U}$, we use $\lambda = 0.25 \times 10^{-4}$.

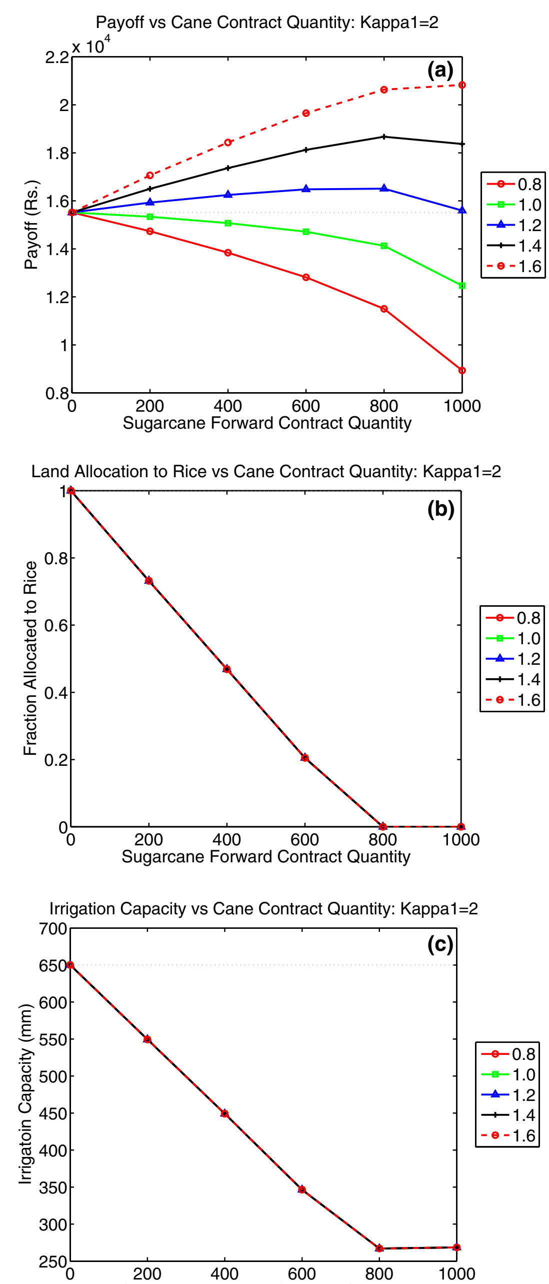
We study the impact of the contract parameters by varying $p^F_{\text{sugarcane}}$ in the range of 80% and 160% of $E[P_{\text{sugarcane}}]$ and also varying $q^F_{\text{sugarcane}}$ between 0 and 1000. The value of $q^F_{\text{sugarcane}} = 0$ corresponds to the case without any contract. Figure 7 summarizes the case of $\kappa^1 = 2$, corresponding to a low value of the irrigation cost, and Figure 8 summarizes the case of $\kappa^1 = 10$, a moderate value of the irrigation cost.

We observe that the payoff to the farmer depends on the contract parameters. For a fixed value of forward quantity $q^F_{\text{sugarcane}}$, the expected payoff in Figures 7a and 8a is unsurprisingly higher if the forward price $p^F_{\text{sugarcane}}$ is higher. It is interesting to note that when the forward price is the same as the expected market price, there is no benefit of the contract to the farmer regardless of the quantity $p^F_{\text{sugarcane}}$. The contract provides the farmer with price certainty, which the risk-averse farmer prefers. At the same time, this exposes the farmer to a greater penalty in case that the yield is not sufficiently high enough to cover the quantity under contract, and thus yield uncertainty becomes a dominating consideration relative to price uncertainty under contract farming. It also forces the farmer to reduce the amount of rice grown. This trade-off may not always be favorable to the farmer, who therefore requires a premium on the forward contract price compared with the expected market price in order to compensate the farmer for the quantity risk associated with sugarcane and the opportunity cost associated with rice.

In addition, these figures show that, for the fixed forward price, the payoff initially increases as the forward quantity $q^F_{\text{sugarcane}}$ increases, but it eventually decreases as the farmer is unable to satisfy the contract quantity and becomes penalized. The payoff is a concave function of the forward quantity. This implies that if the farmer is unsure about the benefit of contract farming, it is safer for him to experiment with it on a smaller scale—with a smaller amount of the contract quantity.

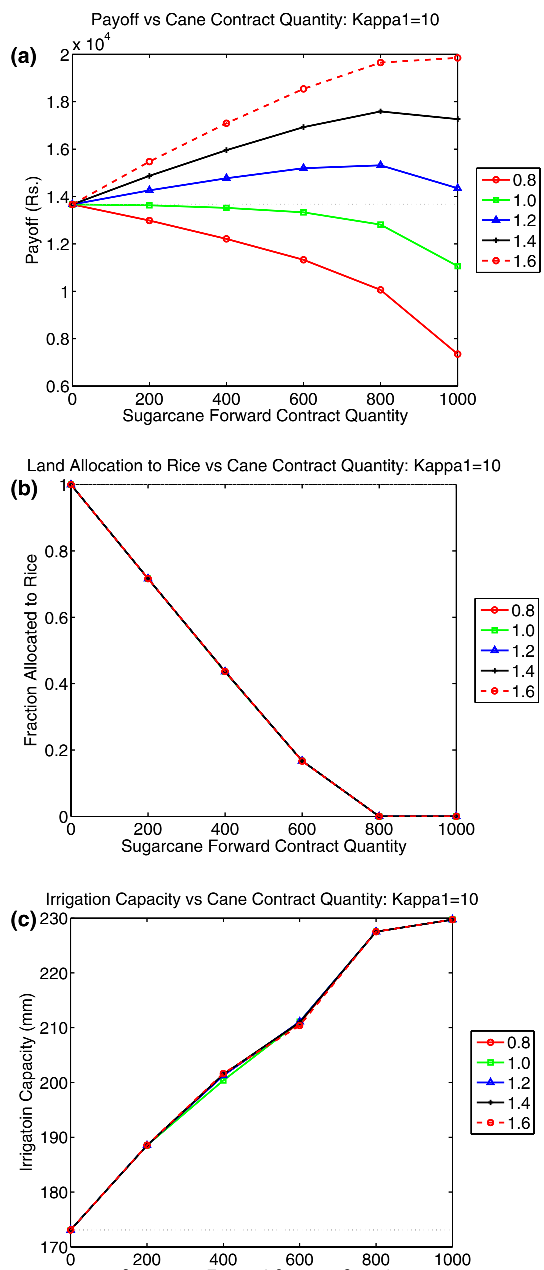
The land allocation decisions are shown in Figures 7b and 8b, which show that the land is to be exclu-

Figure 7 **Contract Farming: $\kappa^1 = 2$ (Model S). Legend Represents the Sugarcane Forward Price as a Multiple of the Expected Market Price, That Is, $p^F_{\text{sugarcane}}/E[P_{\text{sugarcane}}]$. $\gamma = 0.25 \times 10^{-4}$. Note that $q^F_{\text{sugarcane}} = 0$ Corresponds to No Contract Farming. (a) Payoff Function Π vs. $q^F_{\text{sugarcane}}$. (b) Crop-Land Allocated to Rice u_{rice} vs. $q^F_{\text{sugarcane}}$. (c) Irrigation Capacity v vs. $q^F_{\text{sugarcane}}$**



sively used for rice when there is no contract, but more and more land is used for sugarcane as the forward contract requires an increased amount of sugarcane. This monotonicity result is again intuitive. We observe that when the contract quantity $q^F_{\text{sugarcane}}$ is small, the area of land allocated to sugarcane is proportional to $q^F_{\text{sugarcane}}$; in other words, we do not see any aggregation or pooling effect on this area. The

Figure 8 Contract Farming: $\kappa^1 = 10$ (Model S). Legend Represents the Sugarcane Forward Price as a Multiple of the Expected Market Price, That Is, $p^F_{\text{sugarcane}}/E[P_{\text{sugarcane}}]$. $\gamma = 0.25 \times 10^{-4}$. Note that $q^F_{\text{sugarcane}} = 0$ Corresponds to No contract Farming. (a) Payoff Function Π vs. $q^F_{\text{sugarcane}}$. (b) Crop-Land Allocated to Rice u_{rice} vs. $q^F_{\text{sugarcane}}$. (c) Irrigation Capacity v vs. $q^F_{\text{sugarcane}}$.



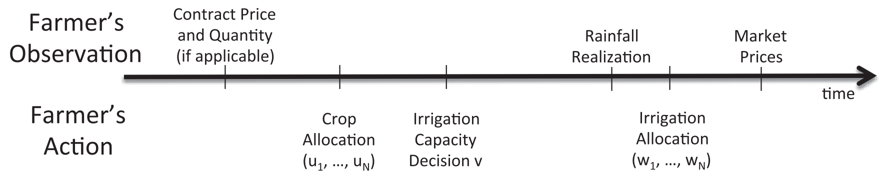
pooling effect on aggregate quantities usually exists because of the reduction of variability from the summation of random variables, but in our case, such a reduction does not exist since the yield rate in one part of the land is perfectly correlated with the yield in another part. When the contract quantity is sufficiently high, the entire land is dedicated to sugarcane.

We note, however, that the farmer's allocation decision is insensitive to the forward price; this phenomenon can be explained by the fact that the value of $p^F_{\text{sugarcane}}$ does not affect the optimal decisions (since $p^F_{\text{sugarcane}}$ appears as a constant in the expression of the profit Π, and the utility function that we use has the CARA property). Since $p^F_{\text{sugarcane}}$, however, affects the farmer's payoff Π, an appropriate choice of this contract parameter will be important in order to entice the farmer to accept this contract in the first place. We deduce that the forward contract quantity is the single-most important predictor for the amount of land in growing sugarcane, provided that the current relative prices between rice and sugarcane remain similar.

Figures 7c and 8c illustrate how the irrigation capacity or quantity depends on the forward quantity. We use the terms "capacity" and "quantity" interchangeably since these two quantities (v and $w_1 + \cdots + w_N$) are the same under Model **S**. We first note that the vertical axis in these two figures are different, and the irrigation capacity is much higher when the associated price κ^1 is lower. It confirms an intuitive result that water usage decreases as a function of the irrigation cost, and thus subsidizing this cost, for example, in the form of subsidized electricity, has an adverse effect on water consumption.

There has been emerging debate regarding whether contract farming contributes to or mitigates water shortage. Some have argued that contract farming increases water usage when farmers shift to crops with high water requirement (Singh 2004), and others have argued that contract farming may potentially reduce water usage through better technology (Sindhu 2010). Figures 7c and 8c further address the question of whether contract farming is beneficial or harmful to water usage. Increasing $q^F_{\text{sugarcane}}$ has two possibly opposing effects on the irrigation capacity planning decision: (i) Since more land is allocated to sugarcane as opposed to rice, and sugarcane requires less water than rice, it follows that the need for irrigation decreases if irrigation is not expensive and (ii) since the contract specifies a higher quantity of sugarcane, more irrigation is required to increase the yield rate of sugarcane. We observe that when irrigation is inexpensive ($\kappa^1 = 2$), the first effect becomes dominant, and the commitment on sugarcane reduces the overall water usage. However, when the irrigation cost is moderate ($\kappa^1 = 10$), the first effect is not

Figure 9 Sequence of Events for Model R (Responsive Irrigation Control)



significant and the second effect is more pronounced; since sugarcane is more sensitive to water input than is rice, the amount of irrigation for sugarcane is less sensitive to price compared to rice. (We have also conducted computations with larger values of κ^1 and the results are qualitatively similar to the case of $\kappa^1 = 10$.) What we can conclude from these observations is that one cannot say whether contract farming unilaterally increases or decreases water usage since the impact of contract farming to the water usage depends on both irrigation cost and contract parameters.

Summarizing the findings of this section, we note that whether the farmer prefers contract farming to no contract farming depends on the contract details. The contract price should be set higher than the expected market price, accounting for the penalty that the farmer needs to pay in case of shortage. The amount of land dedicated to the contract crop depends on the contract quantity, but appears to be insensitive to the contract price. While the impact of contract farming on water usage depends on contract parameters, any subsidy given to the irrigation cost increases water usage.

4. Extension: With Responsive Irrigation Control (Model R)

In this section, we study the sensitivity of our analysis and findings with respect to how we modeled irrigation. In particular, while it was assumed in section 2 that irrigation decisions are made before rainfall is realized, we now consider the case where this decision is made after the farmer obtains a perfect rainfall forecast (see Figure 9). Thus, the farmer knows the exact quantity of rainfall before he starts making decisions on how the irrigated water will be used. The development and use of long-range climate forecasts is a very active area (e.g., Li et al. 2008, Shukla and Mooley 1987, Shukla 1998, for empirical evidence and discussion on how certain aspects of the climate can be predicted). Rather than considering the specific attributes of such forecasts, here we consider the limiting case where a perfect forecast may be available. We refer to this model as the *Model with Responsive Irrigation Control*, or Model **R**. We formulate this model as a two-stage stochastic optimization problem, where the recourse decision in the second stage depends on the realization of rainfall. Our formulation approach and the emphasis on rainfall are consistent with the multi-stage model of Maatman et al. (2002) This section is motivated by an observation that a more realistic model of irrigation should be more dynamic, taking into account the evolution of both rainfall and irrigation throughout the season, but such a model would be more difficult to analyze. Since irrigation precedes rainfall in Model **S**, we consider an alternative model where irrigation follows rainfall and investigate whether the results of earlier sections continue to hold in this case.

4.1. Description

We now present the description of Model **R** by emphasizing the difference of this model from Model **S**. The sequence of events are given as follows.

(1A) At the beginning of the year, the farmer makes a crop allocation decision $(u_1, \ldots, u_N)$, where $u_1 + \cdots + u_N = 1$, and the irrigation capacity decision $v \geq 0$.

(1O) The value of rainfall R is realized, and we denote the realized value by r.

(2A) The farmer makes the irrigation water usage decision $(w_1(r), \ldots, w_N(r))$ where $w_1(r) + \cdots + w_N(r) \leq v$. (While this decision also depends on $(u_1, \ldots, u_N)$ and v, we suppress this dependence in our notation for expositional simplicity.)

(2O) The market prices of crops are realized.

The definition of the payoff function in Model **R** is similar to the corresponding definition in Model **S**, except that the irrigation water usage vector $(w_1, \ldots, w_N)$ needs to be replaced by $(w_1(r), \ldots, w_N(r))$, which is a recourse decision made after the observation of the rainfall quantity r. For Model **R**, it is convenient to introduce $y_i(r)$, which is defined by $r + w_i(r)/u_i$. Then, the yield rate of crop i is $\gamma_i(y_i(r))$, and the total yield of crop i is $x_i = u_i \cdot \gamma_i(y_i(r))$. We can also write the irrigation cost in terms of $(y_1(r), \ldots, y_N(r))$:

$$\mathcal{C} = \mathcal{C}(v, y_1, \ldots, y_N) = \kappa^1 \cdot v - \kappa^2 \cdot r + \kappa^2 \cdot \sum_{i=1}^{N} u_i \cdot y_i(r),$$

which follows from $w_i(r) = u_i \cdot (y_i(r) - r)$ and $u_1 + \cdots + u_N = 1$.

4.2. Two-Stage Stochastic Programming Formulation

In presenting the mathematical formulation for Model **R**, it is convenient to use $\mathbf{y}(r) = (y_1(r), \ldots, y_N(r))$ instead of $\mathbf{w} = (w_1(r), \ldots, w_N(r))$. Note that for fixed r and u_i, $y_i(r) = r + w_i(r)/u_i$ is a one-to-one transformation of $w_i(r)$. In the following formulation, $\mathbf{y}(r)$ is the second stage decision vector (after the rainfall realization r) while $\mathbf{u} = (u_1, \ldots, u_N)$ and v are the first-stage decisions.

$$\text{(SP-R)} \quad \max_{\mathbf{u},v} \; E_{R,\mathbf{P}}[\mathcal{U}(\mathcal{R}(u_1 \cdot \gamma_1(y_1(R)), \ldots, u_N \cdot \gamma_N(y_N(R))) - \mathcal{C}(v, y_1(R), \ldots, y_N(R)))]$$
$$\text{s. t.} \quad \sum_{i=1}^{N} u_i \cdot (y_i(r) - r) \le v \quad \text{for each}$$
r in the support of R
$$u_1 + u_2 + \cdots + u_N = 1$$
$$v \ge 0, u_i \ge 0, y_i(r) \ge r \quad \text{for each } i = 1, \ldots, N.$$

4.3. Analysis: An Overview

While we have presented a two-stage stochastic programming formulation for the farmer's decisions in section 4.2, we now provide a sketch of how to solve this model efficiently. The details of this analysis can be found in Appendix B. We first consider the second stage problem, followed by an analysis of the first stage problem. We show that the problem at each stage can be written as a convex program.

Optimal Irrigation Usage Decision (Second Stage). In the second stage, the farmer decides the recourse decision $\mathbf{y}(r)$. At this time, the first stage decisions, $\mathbf{u}$ and v, have already been made, and the realized rainfall r has been observed. Let $\Pi(\mathbf{u},v,\mathbf{y}(r))$ denote the objective function of (SP-R) condition on the rainfall r, that is,

$$\Pi(\mathbf{u}, v, \mathbf{y}(r)) = E_{\mathbf{P}}[\mathcal{U}(\mathcal{R}(u_1 \cdot \gamma_1(y_1(r)), \ldots, u_N \cdot \gamma_N(y_N(r))) - \mathcal{C}(v, y_1(r), \ldots, y_N(r)))],$$

where we recall

$$\mathcal{C}(v, y_1(r), \ldots, y_N(r)) = \left[\kappa^1 \cdot v - \kappa^2 \cdot r + \kappa^2 \cdot \sum_{i=1}^{N} u_i \cdot y_i(r)\right].$$

Then, the second stage problem can be written as follows:

$$\text{(SP-Ra)} \quad \phi(\mathbf{u}, v) = \max_{\mathbf{y}(r)} \Pi(\mathbf{u}, v, \mathbf{y}(r))$$
$$\text{s. t.} \sum_{i=1}^{N} u_i \cdot y_i(r) \le r + v$$
$$y_i(r) \ge r \quad \text{for each } i = 1, \ldots, N.$$

It can be shown that the above problem is a convex programming problem with linear constraints (Theorem 5 in the Appendix). In the special case of the risk neutral farmer, the first order condition provides a easy-to-understand characterization of the optimal irrigation policy that equalizes the marginal benefit of irrigation for each crop (Proposition 6).

Optimal Crop Allocation and Irrigation Capacity Decisions (First Stage). In our discussion of (SP-Ra), we discussed how to solve the second stage decisions $\mathbf{y}(r)$ while treating the first stage decisions as fixed. For fixed $(\mathbf{u},v)$, recall that $\varphi(\mathbf{u},v)$ is the optimal value of (SP-R) given that the second stage decisions are made optimally. In this section, we study the problem of making the optimal first-stage decisions, which consist of the crop-land allocation $\mathbf{u} = (u_1, \ldots, u_N)$ and the irrigation capacity v, that is,

$$\text{(SP-Rb)} \quad \max_{\mathbf{u},v} \left\{ \phi(\mathbf{u}, v) \middle| \sum_{i=1}^{N} u_i = 1, \; v \ge 1 \right\}.$$

We can show that the objective function that we maximize in (SP-Rb) is jointly concave in the decision vector $(\mathbf{u},v)$ (Theorem 8). This structural result is useful in deciding globally optimal crop-land allocation and irrigation capacity investment Model **R**. We prove this concavity result by first showing the joint concavity of φ—note that this result does *not* follow immediately from standard results and requires a careful construction of the recourse vector.

4.4. Numerical Results

Recall that in section 3 we have reported computation results and insights based on Model **S**. A question that naturally arises at the end of section 3 is whether the results are artifacts of a particular irrigation model that we adopted or whether they are robust with respect to how irrigation is modeled. To answer this question, we have conducted a similar set of experiments for Model **R**, a more dynamic and alternative model of irrigation.

To summarize our findings, the results that we obtain for Model **R** are similar to those reported for Model **S**. In particular, we generated plots that are analogous to 5–8, and the corresponding plots are very close. Each of the three subplots in each figure, especially the total amount of payoff to the farmer, does not seem to be affected by whether irrigation usage decision is a recourse action or not. This implies that the value of flexibility in this recourse action does not have a first-order impact on the payoff. Furthermore, this observation reinforces the insights (that we have identified in section 3) to be robust and insensitive to how we modeled irrigation—since a more realistic dynamic version of irrigation usage lies between the two extremes that we modeled (Models **S** and **R**).

One issue that we further investigate for Model **R** is the distinction between the irrigation capacity and irrigation usage and how it impacts irrigation decisions. Since Model **R** is dynamic, it is possible to reserve a certain level of capacity v, but the amount the farmer uses $w_{\text{rice}} + w_{\text{sugarcane}}$, may in fact be smaller than the capacity, depending on rainfall. (In contrast, under the static model of Model **S**, all irrigation decisions are made before rainfall realization and the irrigation capacity equals usage.) Figure 10a shows the irrigation capacity for Model **R** when the irrigation capacity cost is low ($\kappa^1 = 2$). This plot has a similar shape as the one for Model **S** (see Figure 7c), but we observe that the irrigation capacity for Model **R** is higher compared to Model **S**, which can be explained by the fact that not all the capacity needs to be used. Furthermore, this difference in irrigation capacity between two models is greater when the forward contract quantity $q^F_{\text{sugarcane}}$ is higher. This is due to the fact that sugarcane has a yield rate that is more sensitive to water input than rice, and therefore there is a greater value of flexibility associated with water usage recourse action. These findings are consistent with the moderate value of κ^1 (Figure 11a).

Since the irrigation usage may not be the same as irrigation capacity, we consider how much of the irrigation capacity is actually used. This is shown in Figures 10 and 1. When $q^F_{\text{sugarcane}}$ is small, most of the irrigation is used, but when $q^F_{\text{sugarcane}}$ is high, there is a sizeable gap. This again is related to the water sensitivity of sugarcane. With high values of $q^F_{\text{sugarcane}}$, the land is entirely allocated to sugarcane, and the irrigation usage decision is made to bring the total water input up to the optimal quantity only since excess water input is harmful. However, with low values of $q^F_{\text{sugarcane}}$, the farmer also grows rice, and any excess water can be used for rice since it requires a large

Figure 10 **Contract Farming: $\kappa^1 = 2$ (Model R). Legend Represents the Sugarcane Forward Price as a Multiple of the Expected Market Price, That Is, $p^F_{\text{sugarcane}}/E[P_{\text{sugarcane}}]$. $\gamma = 0.25 \times 10^{-4}$. Note that $q^F_{\text{sugarcane}} = 0$ Corresponds to No Contract Farming. (a) Irrigation Capacity v vs. $q^F_{\text{sugarcane}}$. (b) Expected Irrigation Usage $\omega_{\text{rice}} + \omega_{\text{sugarcane}}$ vs. $q^F_{\text{sugarcane}}$.**

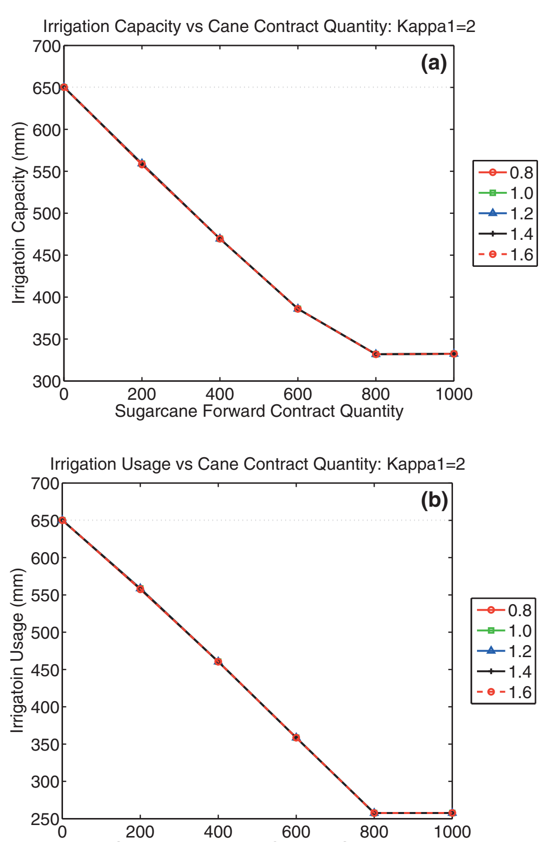
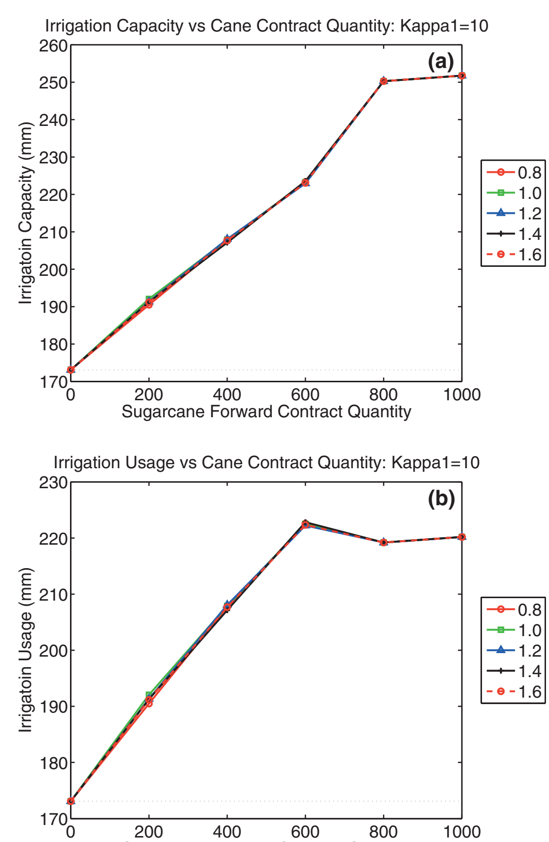


Figure 11 **Contract Farming: $\kappa^1 = 10$ (Model R). Legend Represents the Sugarcane Forward Price as a Multiple of the Expected Market Price, That Is, $p^F_{\text{sugarcane}}/E[P_{\text{sugarcane}}]$. $\gamma = 0.25 \times 10^{-4}$. Note that $q^F_{\text{sugarcane}} = 0$ Corresponds to No Contract Farming. (a) Irrigation Capacity v vs. $q^F_{\text{sugarcane}}$. (b) Expected Irrigation Usage $\omega_{\text{rice}} + \omega_{\text{sugarcane}}$ vs. $q^F_{\text{sugarcane}}$.**



amount of water but is not as sensitive to water input. This observation leads to a useful implication for a policymaker or anyone else interested in reducing the amount of water usage through contract farming in sugarcane. To this end, it is preferable to set up a contract that specifies the forward contract quantity $q^F_{\text{sugarcane}}$ to be high enough to induce the farmer to grow sugarcane exclusively. Otherwise, if the farmer grows both sugarcane and rice, the farmer may increase his investment in irrigation in order to control irrigation for water-sensitive sugarcane, and any residual excess capacity will be used for rice, thereby increasing total irrigation usage. This occurs since the water usage cost κ^2 is low compared to the water capacity cost κ^1, which is the case of the Indian subcontinent context. We note that the policymaker's preference for a large contract is in contrast to the farmer's preference for a smaller contract that we discussed in section 3.

In this section, we have investigated whether a different model of irrigation would change our earlier findings, and it turns out that irrigation flexibility does not introduce substantial changes to our results. We observe that when the irrigation usage decision is made after the rainfall is realized, not all capacity that was planned for will be actually used. When the water usage cost is relatively inexpensive, however, we see a tendency that if the irrigation capacity planned for a water-sensitive crop is not used for this crop, it may end up being used for a less water-sensitive crop.

5. Concluding Remarks

In this study, we have developed a simple and stylized model for the farmer's decision under the rainfall and price uncertainty that explicitly incorporates the irrigation investment and usage decisions. We have identified several structural properties (e.g., the concavity of objective functions) that not only produce managerial insights but also allow the numerical computation for the optimal solution to be uncomplicated. We have studied the relationship between crop planning and irrigation management through numerical analyses based on historical agricultural data from the Ganganagar district of India and illustrated that contract farming can be used to increase the profit of the farmer as well as to encourage the responsible usage of water, though it is dependent on the contract and cost parameters.

In particular, we have shown that the farmer under forward contracts is, and this means shielded from price uncertainty, but is exposed to a greater risk of quantity or yield uncertainty, Because of this tradeoff, the contract price may need to reflect a premium on the market price for the farmer to accept the contract. The farmer's payoff is concave in the contract quantity, and this means the farmer may start the contract farming relationship with a small contract to test its benefit. With regard to water usage, contract quantity seems to be the single most important factor in its impact on water usage, but whether this impact is positive or negative depends on cost parameters. Therefore, the impact of contract on water cannot be simply stated—although a carefully designed contract in certain settings can reduce water usage. Finally, water usage may increase with diversified farming in which the farmer grows multiple crops, and this increase is due to the increased value of irrigation capacity when it is flexible—the irrigation capacity can be used for any subset of crops. Thus, with diversified farming, one can obtain a better solution by allowing the irrigation cost to depend not only on the capacity investment but also on actual water usage.

There are several interesting research thrusts related to this study. These are outlined below and reflect possible directions in our current research within this area.

First, we need to consider the buyer's problem and its interface with the farmer's problem. In this study, our focus was primarily on the farmer since understanding his objectives and behavior is a prerequisite to what kind of contract the buyer can offer, but the farmer's perspective has not been satisfactorily studied in the literature. Potentially, the buyer will have a basket of crop needs and superior information relative to the farmer as to potential market prices, rainfall forecasts, and technologies. Also, the buyer may intend to contract with potentially hundreds or thousands of farmers in diverse geographical areas to meet the target product demands. The asymmetry in information and in capital resources could translate into predatory contract price setting. On the other hand, the local market prices potentially provide a balancing mechanism. If the buyer procures a large fraction of the local product, then he will change local market prices, possibly making an adverse impact on participation or default incentives of the farmer. There is also opportunity for cooperative interaction between the farmer and the buyer. Both face climate and market price risks, but in somewhat different ways. Geographical diversification of procurement could reduce the buyer's climate risk, but increase transaction costs. The farmer may not have the capital or the financial resources to afford crop insurance or to purchase irrigation equipment or fertilizer to the optimal production level. The buyer could then negotiate in terms of optimizing his expense across geographical risk pooling or building sustainable capacity and goodwill with specific farmer or farmer groups through the needed capital. The insurance case is interesting because the geographical risk pooling (perhaps to a limited degree) would still provide the buyer with the ability to get reinsurance cover reflecting the reduced risk, as opposed to the retail insurance policy the farmer is constrained to buy.

Second, the opportunity to reduce the stochastic elements of the problem could come either through investment (perhaps social or corporate) in forecasting systems (climate or commodity prices) and the supporting monitoring programs or through the optimization of water storage, irrigation, and the fertilizer mix. Crop diversification is also a social and individual goal that possibly enters as a natural linear constraint into such a formulation either from a policy perspective for the corporation or government or even for the farmer considering subsistence as well as revenue goals. The models as formulated could be used to explore the trade-offs in risk reduction by different means with a parametric evaluation of the crop diversification and budgetary constraints. In this case, one would also need to consider objective functions or goals that reflect elements of risk that are not captured well by the expected value analysis considered in this study.

Third, in a setting such as India, the government offers a variety of subsidies for water inputs, provides support prices for certain crops deemed to be in the interest of national security, and provides relief from flood and drought losses. While many of these program elements are politically expedient, some are needed for risk management (though not necessarily in the most optimal sense) of rural livelihoods, and the net effect on environmental and water resource sustainability is perceived as negative due to the resulting farmer decisions to unsustainably pump groundwater to grow rice or other crops in regions and seasons that are climatologically unsuitable for such crops. Here, the question is whether the private sector, through contract farming, can effectively compete with or help re-shape government programs to promote more sustainable water, crop, and rural livelihood outcomes at the national scale. This requires multi-scale and multi-agent modeling in the same framework as initiated here, and a broader set of questions needs to be considered.

For the deployment of our model in a field setting (our current work focuses on India), the underlying assumptions of the model need to be calibrated for the weather and soil conditions of the specific region under study, and furthermore, additional features and constraints should be added to the model (e.g., the trade-off between water use and fertilizer application that are generally well studied and hence were implicitly parameterized in the current application). Regarding contract farming, we have illustrated its potential benefits using forward contracts assuming that the details of the contracts are exogenously given, and we are currently investigating the negotiation process of determining specific parameters of the forward contract and other types of contracts as well as the possibility of contract enforcement and renegotiation (Plambeck and Taylor 2007). Overall, we envision that the increased role and participation of private corporations, in collaboration with policymakers and farmers, can address and resolve the problems of public interest (namely, agriculture and water), provided that competing and incongruent objectives are properly balanced and managed with appropriate incentives.

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Note

[1]In this study, the amount of water is measured not in terms of volume, but rather in terms of the “height” when it is spread over the unit area. We use this convention to be consistent with the unit of rainfall, which is also measured in height.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix A.
Appendix B.