IIMB Management Review (2012) 24, 187—202

available at www.sciencedirect.com

SciVerse ScienceDirect

journal homepage: www.elsevier.com/locate/iimb

Contract farming with possible reneging in a developing country: Can it work?

Woonghee Tim Huh [a,*], Stergios Athanassoglou [b], Upmanu Lall [c]

[a] *Sauder School of Business, University of British Columbia, Vancouver, BC V6T1Z4, Canada*
[b] *Euro-Mediterranean Center for Climate Change, Fondazione Eni Enrico Mattei, 20123 Milan, Italy*
[c] *Columbia Water Center and Department of Earth and Environmental Engineering, Columbia University, New York, NY 10027, USA*

Abstract We consider a processed-food manufacturer that faces uncertain exogenous demand and procures a farm crop either from the outside market or from local farmers via contract farming. The contract price is determined at the beginning of the season when the market price is still uncertain. When the market price is realised, we allow the farmer the possibility of reneging from the contract, which occurs if the market price is sufficiently high. We show that granting farmers the option of reneging on the contract may improve the manufacturer's expected profit, and identify the conditions under which such an improvement can be expected.

Introduction

The world leaders at the Millennium Summit of the United Nations adopted the Millennium Declarations in September 2000, and committed their nations to the cause of reducing extreme poverty in the world and to the setting of time-bound targets. To this effect, they have identified and agreed on the Millennium Development Goals (MDGs) containing eight specific goals, the first of which is the *eradication of extreme hunger and poverty* (UN Millennium Project, 2005). According to a report by the United Nations Food and Agriculture Organization (2002) on the long-term development of world food and agriculture for the next three decades, serious food shortages are expected to persist at regional and local levels. This report also suggests that appropriate interventions as well as increased international agricultural trade may moderate the magnitude and the effect of the supply shortage problem. Since a large proportion of the population in developing countries is rural and engaged in agriculture, the above-mentioned Millennium Development Goal is closely related to the profitability and the crop yield management of small-scale farmers in those countries.

The most common approach for addressing the problems of the global and national food security as well as the rural poverty and equity issues has been government intervention — in the forms of energy subsidies, virtually free access to irrigation, crop support, and grain procurement. While such

* Corresponding author.
E-mail addresses: tim.huh@sauder.ubc.ca (W.T. Huh), athanassoglou@gmail.com (S. Athanassoglou), ula2@columbia.edu (U. Lall).
Peer-review under responsibility of Indian Institute of Management Bangalore

Production and hosting by Elsevier

0970-3896
http://dx.doi.org/10.1016/j.iimb.2012.06.003

interventions are well intentioned and often provide short-term relief to the farmers, their outcomes have been at best mixed in terms of providing long-term sustainable solutions. The problem with many government-initiated programmes is that engendering changes often involves a long and challenging political process. Meanwhile, the private sector may play an increasing role in improving the overall efficiency in the agriculture industry and the agri-business industry. This improvement could be achieved through non-traditional methods such as investing in infrastructure, adopting new technologies like drought-resistant high-yield strains, and successfully implementing crop diversification strategies. Corporations often have better access to models for commodity pricing and weather forecasting, that could be of potential benefit to local farmers.

One of the avenues of forming a symbiotic partnership between private corporations and farmers is through contract farming. Under this arrangement, the corporation requests farmers to plant a specific crop, and purchases it according to the initially agreed upon terms of the contract. The corporation benefits from this relationship since it can secure the quantity and quality of the desired crop which may not be sufficiently available in the area. In addition, it possibly reduces or eliminates certain expenses associated with transportation, storage and spoilage. The farmer often earns a higher and stable revenue, and obtains access to capital, technology and information that would increase his yield and that would otherwise not be easily obtainable (Rehber, 1998).

Despite several potential advantages of contract farming to both the corporation and the farmers, one of the problems associated with its implementation is that the farmers may sell outside the contract (Food and Agriculture Organization, 2001). This phenomenon, referred to as *extra-contractual marketing*, has caused some corporations either to monitor the farmers more closely or to abandon contract farming altogether. It is exacerbated by the fact that, in many developing countries, it is difficult to enforce the terms of the contract due to the complexity of the legal system (Reardon & Barrett, 2000; Reardon & Berdegue, 2002; Runsten & Key, 1996). It is reported that the first large-scale demonstration of contract farming in India, initiated by Pepsi Foods in India, "bound the farmer morally rather than legally" (Khairnar & Yeleti, 2005).

Motivated by this concern, we consider a model of contract farming which explicitly accounts for the possibility of the farmer's reneging. We model the system with a single firm and multiple homogeneous farmers. The firm is a manufacturer of a processed food product that is based on a farm crop. On the demand side, she faces exogenous uncertain demand for the product. On the supply side, we are particularly interested in the case in which the manufacturer newly builds a production capacity in a developing country where the raw material for non-staple crops has not been traditionally available in large quantities, and she offers contracts to the farmers for the crop — the rationale for the contract farming based on this has been pointed out by Key and Runsten (1999). We also allow that the manufacturer may also purchase the crop from the external market that exhibits a volatile price. Since the market system is not well established in developing countries, the transportation (or transaction) cost cannot be ignored, and we include it in our model. The farmer, if entered into the contract, plants the crop and is supposed to deliver his yield to the manufacturer, but we allow the possibility that the farmer may renege on the contract and sell the crop directly to the market if the market price is attractive compared to the contract price. We analyse this model, and compare it to (1) the case of no contract; and (2) the case of a contract without reneging.

From our interaction with a multinational corporation in the food processing industry that has been engaged in contract farming, we identify three primary sources of uncertainty in forecasting and in planning for future operations: (i) the commodity prices of farm products that serve as raw materials for its manufacturing operations, (ii) the amount of supply of these products which depends on crop planting as well as crop conditions that are largely affected by the weather and water availability, and (iii) consumer demand or market conditions which follow population and demographic trends. We incorporate these types of uncertainty into our model.

In this paper, we take the view of the manufacturer who wants to maximise her expected profit. A natural question that arises is the following: Which of the three contract models will result in the highest expected profit for the manufacturer? We show that the contract can be beneficial to the firm even when the farmers may renege. Moreover, while the dominance between the two contracts (one with reneging and one without) cannot be established, we show, somewhat counter-intuitively, that the farmer's reneging option may benefit the manufacturer. We then identify under which conditions such a benefit can be expected. For instance, in the special case of deterministic demand and yield and risk-averse farmers, we formally prove that the reneging option results in a greater payoff for the manufacturer. Thus, the farmer's reneging is not simply to be avoided or prevented; if this possibility is properly managed and leveraged, it may increase the manufacturer's bottom line.

Literature review

There is a large body of literature on the application of operations research and operations management to the farmer's crop planning — see, for example, Glen (1987), Lowe and Preckel (2004) and Huh and Lall (2008). When the yield is uncertain, there is a risk of supply and demand mismatch and such a risk is managed and mitigated by the capacity-production decision that accounts for such uncertainty (Allen & Schuster, 2004; Kazaz, 2008), or by the second production opportunity (Jones, Kegler, Lowe, & Traub, 2003; Jones, Lowe, Traub, & Kegler, 2001; Jones, Lowe, & Traub, 2002; Kazaz, 2008). All these papers are presented in the context of a single decision maker and do not consider any contract structure.

Much of the existing research on contract farming is empirical or case-based (e.g., Runsten and Key (1996) and Warning and Key (2002)). There are a limited number of recent analytic models for the use of contracts in the agriculture business. Burer, Jones, and Lowe (2008) examine two commonly used contracts in the agricultural seed industry between the seed supplier and the retailers, called the "pure bonus scheme" and "mixed scheme", and

He and Zhang (2008) study several risk sharing contracts under random yield in agriculture. However, these models do not directly address the contract farming problem between a firm and many farmers.

We mention several analytical works related to contract farming. Ghosh and Raychaudhuri (2010) analyse how contract farming is used by the farmer as a protection against price risk in India, where proper insurance system does not exist. Delpierre (2005) considers the possibility of multiple buyers competing with each other. Patlolla (2010) studies the sugarcane contract farming in India, and shows that the buyer can ensure higher quality of crops by using an alternative contract scheme. Thome and Sexton (2007) study the possibility of reneging by the buyer not the farmer.

Contract farming is similar to a well-known concept in operations management called *contract manufacturing*, where the contract manufacturer and the original equipment manufacturer enter into a contract based on core competency (Plambeck & Taylor, 2005; Ülkü, Toktay, & Yücesan, 2007). In the case where the contracts are court-enforceable, Plambeck and Taylor (2006, 2007c, 2007d) show that the use of informal agreements (informal contracts) can attain the optimal outcome, especially in the environment of repeated interaction where the prospect of future business depends on current actions (see also Domberger (1998)). In contrast to this literature, we model a single-period interaction since the farmer may be too poor or too myopic to consider the benefit of future interactions. While the reneging opportunity in our model is one-sided (only the farmer may renege on the contract in our model), we mention that Plambeck and Taylor (2007a, 2007b) consider the case where multiple parties decide to renegotiate depending on the outcome of uncertain events. This assumption is consistent with the realities of the contract farming in developing countries (Brennan, 2004).

A number of papers in the economics literature have addressed the issue of bargaining power between the firm (buyer) and farmers (suppliers). It has been noted that the farmer's share of the total profit increases as his outside option (alternative production possibility) improves and as his relative bargaining power increases (Swinnen & Vandeplas, 2007). Since there are multiple farmers but a single firm, the firm is endowed with monopsony power (Sivramkrishna & Jyotishi, 2008). Salas (2009) proposes the formation of a "bargaining group" to shift the market power from the buyer to the sellers. In our paper, we model the presence of the outside option for the farmer, but we assume that the buyer has complete bargaining power and maximises her expected profit.

A distinguishing feature of our paper is that we model contract farming in the context of a larger supply chain. We consider not only the relationship between the farmers and the manufacturer, but also the manufacturer's production decision facing uncertain demand. Our modelling of demand places contract farming in the framework of the newsvendor problem, well known in the operations management literature. We are particularly interested in the conditions under which the manufacturer would benefit from a contract that cannot enforce the farmer's compliance. This problem is of considerable interest to firms operating in developing countries such as India.

Organisation

The remainder of the paper is organised as follows. We first present a detailed description of the model. We then analyse the case where both the yield and demand distributions are deterministic. We include and discuss both analytical and numerical findings. Then we extend our results to the general model with stochastic yield and demand.

Model

We consider a manufacturer in the food processing industry, who takes a farm crop as a raw material and transforms it into a finished product (e.g. food). This product has traditionally been unavailable in the area; however, its demand is now emerging (due to industrialisation and globalisation), and the manufacturing facility is set up to target such a trend in demand. Let D denote a random variable representing the quantity of demand for the product. (In this paper, we use lower case variables to represent the realised values of corresponding upper case random variables.) Let r denote its selling price, which is constant and exogenously given.

We assume that the farm crop that the manufacturer requires has also been unavailable in the area (due to the incompatibility of the crop with the traditional diet or the magnitude of the manufacturer's requirement), and that there are two ways of securing the supply of the crop. We describe them in the following two paragraphs.

(i) The manufacturer can procure the required crop by importing it from an outside market, in which case she is subject to an additional transportation cost (or transaction cost), and she is also exposed to the price uncertainty of the crop. Let K_M be the unit transportation cost from the market to the manufacturer (which is a constant), and let P denote the random variable for the market price of the crop. The value of P is initially stochastic, and it is realised at the end of season when the crop is harvested.

(ii) Alternatively, the manufacturer can encourage the local farmers to grow the farm crop by offering them contracts. We model the contract between the manufacturer and a farmer such that both parties agree that the entire harvest of the crop will be sold at an agreed-upon price $\hat{p}$, which we refer to as the contract price. Note that this contract is simple and specified by a single parameter (price), an attractive feature in the rural economies of developing countries. In our model, the manufacturer offers the contract price, which each farmer may accept or reject. The manufacturer also decides the number of farmers to whom she offers the contract. (The number of farmers in this paper is essentially a proxy for the size of the contract representing the amount of land under the contract.)

In our model, there are multiple farmers and they are all indistinguishable from one another; thus, it suffices to consider a representative farmer who possesses a unit area of land. We let C denote the farmer's opportunity cost for signing the contract, which corresponds to the profit from the traditional use of the land. (We allow that C may be random.) Any farmer who signs the contract will plant the crop. Let Y denote the random variable representing the yield of the crop per unit area. (In the farming setting, the randomness in yield is typically due to the amount of rainfall and the use of fertilizers — see Li, Li, and Li (2004) for the case of wheat.) We incorporate into our model the possibility of reneging by farmers. Let K_F be the unit transportation cost from any farmer to the market, which is a constant. If a farmer sells the crop to the market, he generates the net revenue of $p - K_F$ per unit, where p is a realised value of P. Thus, if the realised market price p is sufficiently high, i.e., $p - K_F > \hat{p}$, then it is profitable for the farmer to renege on the contract and sell his crop to the market. The phenomenon of reneging is not uncommon in developing countries since the legal systems are too complex for the contracts to become enforceable and the farmers are too myopic to appreciate the benefit of the long-term relationship with the manufacturer.

In contrast to the farmers, the manufacturer in our model does not have the flexibility to renege on the contract. The manufacturer often represents a large (possibly multinational) corporation, and reneging is too costly in terms of tarnishing her reputation and risking adverse political repercussions. However, if a farmer reneges on the contract, the manufacturer may choose to make a counteroffer, and this counteroffer will be accepted by the farmer if it is at least $p - K_F$, and rejected otherwise. We allow that the manufacturer may choose to generate counteroffers to only a subset of the farmers who have reneged.

We present the sequence of events in detail.

(i) The manufacturer decides the contract price $\hat{p}$ and the number of farmers q to which the contract is offered. Then, each farmer either accepts or rejects the contract. If the contract is accepted, the farmer plants the crop in his farmland; if rejected, he saves the opportunity cost C, which is realised at this point.
(ii) The yield Y is realised, and each farmer harvests y units of the crop, where y is a realised value of Y. The market price P is also realised, and its realised value is denoted by p.
(iii) Each farmer who has accepted the contract decides whether to honour the contract or to renege on the contract. If he honours the contract, the manufacturer buys y units of the crop from him at the price of $\hat{p}$ per unit. Let x denote the total amount of the crop the manufacturer possesses at this point. (Clearly, $x \le y \cdot q$.)
(iv) The manufacturer decides the production quantity z for the product. For simplicity, we assume that one unit of the crop is needed to produce one unit of the product, and that any production cost other than the price of the crop can be ignored. To any farmer who has reneged on the contract, the manufacturer has an option of making a counteroffer (with a new price), which may or may not be accepted. If a farmer does not sell his crop to the manufacturer, then he sells it to the market generating the revenue of p per unit and incurring the transportation cost of K_F per unit. The manufacturer purchases the shortage quantity of the crop, if any, from the market at p per unit, incurring the transportation cost of K_M per unit.
(v) Demand D for the product is realised, and the manufacturer satisfies realised demand d to the extent possible. The sales quantity for the product is $\min\{d, z\}$, and the revenue to the manufacturer is $r \cdot \min\{d, z\}$. Any unsold product is scrapped at no additional value or cost (Table 1).

While the manufacturer can purchase the crop from the market, we note that she does not sell excess crop to the market. This assumption, based on our interaction with multinational manufacturing firms operating in a developing country, plays an important role in our paper, and such a phenomenon is reasonable (a) when the distribution logistics of the manufacturer are dedicated to her product and are not suited for the farm crop — for example, unable to circumvent perishability of degradation in quality over time, (b) when the manufacturer desires to cultivate a symbiotic relationship with the farmers by avoiding direct competition as sellers in the same market, and (c) when government regulation or complexity of local social structure poses the manufacturer (often a multinational firm) a barrier of entry in the domestic raw material market

Table 1 Model parameters and variables.

Stochastic parameters	
D	Demand for finished product produced by firm
P	Market price of crop
Y	Crop yield
C	Opportunity cost to farmer of accepting contract
Deterministic parameters	
r	Selling price of product
K_M	Unit transp. cost to manufacturer of procuring crop from market
K_F	Unit transp. cost to farmer of selling crop to market
Decision variables	
$\hat{p}$	Contract price offered
q	Number of farmers offered a contract
x	Total crop quantity acquired using contracts
z	Production quantity of finished product

(Nayak, 2000). As a consequence, since we assume no cost of production other than the cost of the crop, it follows that the production quantity of the manufacturer will be at least the amount of the crop she collects from the contracted farmers in step (iii), i.e., $z \geq x$.

We are now ready to discuss the objective functions of the farmer and the manufacturer. In our model, each farmer is risk-averse, and we assume that his economic benefit is represented by a utility function $\mathcal{U}(\cdot)$ that is concave increasing in his net profit; he wants to maximise his expected utility. In contrast, we assume that the manufacturer is risk-neutral and that she maximises her expected profit. In our discussion, it is helpful to note that in step (iv) the farmer will accept any counteroffer that is at least the price he can fetch from the market minus the transportation cost, and thus the profit-maximising manufacturer sets her counteroffer price at $p - K_F$. Then, the farmer's expected utility is $E_C[\mathcal{U}(C)]$ if he rejects the contract; otherwise, his objective is to maximise

$$\Pi_F = E\left[\mathcal{U}\left(Y \cdot \max\{\hat{p}, P - K_F\}\right)\right].$$

Returning to the manufacturer, he purchases x units at the contract price $\hat{p}$, and he can purchase up to $Y \cdot q - x$ units by making counteroffers at the price of $P - K_F$. Any additional unit can be bought from the market at the net cost of $P + K_M$ per unit. Since the amount of her sales is min $\{D, z\}$, it follows that the manufacturer's profit is given by

$$\begin{aligned} \Pi_M = & -x \cdot \hat{p} - E\Big[(\min\{z, Y \cdot q\} - x) \cdot (P - K_F) \\ & - [z - Y \cdot q]^+ \cdot (P + K_M) + r\min\{D, z\}\Big]. \end{aligned} \quad (1)$$

In our model, there are two types of uncertainty, one involving price (the market price P and the opportunity cost C), and the other involving quantity (crop yield Y and product demand D). We first address in the following section the case where uncertainty arises only from the market price, and then we consider both types of uncertainty.

Deterministic yield and demand

In this section, we consider the case where the only stochastic variables are the market price P and the opportunity cost C. Thus, both Y and D are deterministic, which we denote by y and d, respectively. While we analyse the behaviour of both the manufacturer and the farmers, the subject of our primary interest is the manufacturer, and we examine the benefit of the contract to the manufacturer despite the possibility that farmers may renege. As a first step, we consider the benchmark model where no contract is offered. Next we present the analysis of our model, and subsequently we study a modified model where the farmers *cannot* renege. Finally we investigate the performance of the contract with reneging by comparing it to the two related models.

A benchmark model: without contract

We first consider as a benchmark the case where no contract is offered. This model is useful not only in quantifying the benefit of contract farming, but also in analysing both types of contracts that follow. Here, the farmer only has the "outside option" which is the opportunity cost. Thus he obtains the expected utility of

$$\Pi_F^{NC} = E[\mathcal{U}(C)].$$

(The superscript NC represents "no contract".) For the manufacturer, since the selling price of her product is r, she purchases the crop from the market only if the net cost $P + K_M$ is below r. Since $D = d$ is deterministic, the manufacturer's expected profit is given by

$$\Pi_M^{NC} = d \cdot E_P[r - (P + K_M)]^+. \quad (2)$$

Contract with reneging: analysis

We now address the original model with reneging described in the section "Model". We solve this model in a backward manner starting with step (v) toward step (i). Since steps (ii) and (v) do not involve any decision, we consider steps (iv), (iii) and (i).

Step (iv): manufacturer's decision for z

We first analyse the manufacturer's decision for the production quantity z in step (iv) of the model. Recall that x represents the amount of the crop that the manufacturer has purchased from the farmers based on the contract, where $x \leq y \cdot q$. From an earlier discussion, z must satisfy $z \geq x$. If this inequality is strict, then the manufacturer needs to procure the additional units either from making counteroffers to the reneged farmers or from the market. Since the revenue that a reneging farmer can fetch from the market is $p - K_F$ per unit, any counteroffer that is at least as good as this price will be accepted; as a result, the manufacturer can procure up to $y \cdot q - x$ units (which is the total reneged quantity) at the price of $p - K_F$ per unit. She can procure the remaining units from the market at a higher cost of $p + K_M$ per unit which includes the transportation cost.

The optimal decision for z depends on the realised value of the market price p for the farm crop (raw material) in comparison to the sales price of the product, r. Recall that in this section we assume that demand d is deterministic. If p is sufficiently low, i.e., $p \leq r - K_M$, then she satisfies all the demand; thus, $z = \max\{d, y \cdot q\}$. If $p \in (r - K_M, r + K_F)$, it is optimal to procure only from counteroffers to reneged farmers where the price of the counteroffer would be $p - K_F \in (r - K_M - K_F, r)$. In this case, since the amount of the crop that the manufacturer possesses and the reneged quantity sum up to $y \cdot q$, it follows that $z = \min\{\max\{d, x\}, y \cdot q\}$. If $p > r + K_F$, then the crop price is prohibitively expensive to make any additional crop purchase, i.e., $z = x$. In summary, the manufacturer's optimal decision in step (iv) is

$$z = \begin{cases} \max\{d, y\cdot q\} & \text{if } p \le r - K_M \\ \min\{\max\{d, x\}, y\cdot q\} & \text{if } p \in (r - K_M, r + K_F] \\ x & \text{if } p > r + K_F. \end{cases} \quad (3)$$

Step (iii): farmer's decision for reneging

We now consider a farmer's decision in step (iii) regarding whether to honour the contract or renege on it. If he honour the contract, he will receive $\hat{p}$ per unit from the manufacturer. If he reneges on the contract, it follows from our earlier analysis that his unit net revenue will be $p - K_F$ regardless of whether the manufacturer makes a price-matching counteroffer, or the farmer sells his crop to the market. (Note that, at this point, the realised value of the market price p is known.) Thus, it is optimal for the farmer to honour the contract if $p \le \hat{p} + K_F$ and renege on it otherwise. As a result, the amount of inventory that the manufacturer receives at the end of step (iii) is

$$x = \begin{cases} y\cdot q & \text{if } p \le \hat{p} + K_F \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Step (i): manufacturer's decision for $\hat{p}$ and q

Since there is no decision involved in step (ii), we now consider step (i), in which the manufacturer sets the contract price $\hat{p}$ and the number of farmers q that she offers the contract. This step takes place before the market price uncertainty is resolved in step (ii). For each farmer, the expected revenue given that he signs the contract and plants the crop is $y\cdot\max\{\hat{p}, P - K_F\}$, which is stochastically increasing in the contract price $\hat{p}$. Thus, a farmer accepts the contract if the expected utility from the contract is at least as high as the expected utility from the alternate option, i.e.,

$$E_P\left[\mathcal{U}\left(y\cdot\max\{\hat{p}, P - K_F\}\right)\right] \ge E_C[\mathcal{U}(C)]. \quad (5)$$

Note that the left-hand size expression, representing the expected utility from the contract, is a nondecreasing function of $\hat{p}$. Thus, the above inequality is equivalent to

$$\hat{p} \ge \hat{p}_{LB}, \quad (6)$$

for some $\hat{p}_{LB}$. Here, $\hat{p}_{LB}$ represents the contract price at which the farmer is indifferent between the contract and the outside option.

We make a few observations on how the farmer's indifference price $\hat{p}_{LB}$ depends on the distribution of the market price P.

- As the market price P increases stochastically, the left-hand-side expression of (5) increases while the right-hand-side remains constant; thus, the threshold price $\hat{p}_{LB}$ can be smaller. More intuitively, as the farmer expects the market price to be higher, his profit from reneging (or from a counteroffer) would be higher. Thus, he is more likely to plant the crop, and the contract price does not have to be as attractive.
- As the market price P becomes more variable, the farmer's indifference $\hat{p}_{LB}$ tends to decrease. This occurs since the contract becomes more appealing as the variability of the market price P increases — it opens the possibility of a higher profit from reneging while the downward risk is protected by the contract price. (We can prove for a risk-neutral farmer for a symmetric price distribution.) Let Λ be a symmetric distribution with mean 0 and standard deviation 1 with a probability density function f, and suppose $P = K_F + \mu + \sigma\Lambda$, where the mean and the standard deviation of P is given by $K_F + \mu$ and σ, respectively. Let $\hat{z} = \hat{p} - \mu$. Then, $E_P[\max\{\hat{p}, P - K_F\}] = \mu + E_\Lambda[\max\{\hat{z}, \sigma\Lambda\}]$. Since the probability density of $\sigma\Lambda$ is $f_\sigma(z) = f(z/\sigma)/\sigma$, we have

$$E_\Lambda\left[\max\{\hat{z}, \sigma\Lambda\}\right] = \int_{-\infty}^{\hat{z}} (\hat{z} - z) f_\sigma(z) dz$$
$$= \sigma \int_{-\infty}^{\hat{z}/\sigma} \left(\frac{\hat{z}}{\sigma} - w\right) f(w) dw = \sigma E_\Lambda\left[\max\left\{\frac{\hat{z}}{\sigma}, \Lambda\right\}\right].$$

This expression is increasing in σ if $\hat{z} \le 0$. We proceed by assuming $\hat{z} \ge 0$.

$$E_\Lambda[\max\{\hat{z}, \sigma\Lambda\}] = \int_{-\infty}^{-\hat{z}} (-\hat{z} - z) f_\sigma(z) dz + 2\hat{z}P[\sigma\Lambda \le -\hat{z}]$$
$$+ \int_{-\hat{z}}^{\hat{z}} (\hat{z} - z) f_\sigma(z) dz = \int_{-\infty}^{-\hat{z}} (-\hat{z} - z) f_\sigma(z) dz$$
$$+ 2\hat{z}P[\sigma\Lambda \le -\hat{z}] + 2\hat{z}P[-\hat{z} \le \sigma\Lambda \le 0]$$
$$= \int_{-\infty}^{-\hat{z}} (-\hat{z} - z) f_\sigma(z) dz + \hat{z},$$

where the second equality follows from $\int_{-\hat{z}}^{\hat{z}} (\hat{z} - z) f_\sigma(z) dz = \int_{-\hat{z}}^{0} (\hat{z} - z) f_\sigma(z) dz - \int_{-\hat{z}}^{0} (\hat{z} + z) f_\sigma(z) dz = 2\hat{z}P[-\hat{z} \le \sigma\Lambda \le 0]$ which holds by the symmetry of f. By using the same argument as above, the above expression is also increasing in σ. This implies that the indifference price $\hat{p}_{LB}$ is decreasing in the standard deviation σ.)

Now, we address the manufacturer's decision for determining both the number of contracts q and the contract price $\hat{p}$. It turns out that there is no reason why the manufacturer should set the contract price $\hat{p}$ any higher than the minimum price $\hat{p}_{LB}$ required to induce the farmers to participate. Furthermore, if a contract is offered to the farmer, then the number of contracts should be just enough to cover the demand for the product. These results are obvious and stated formally in the following proposition.

Proposition 1. *Suppose that both $D = d$ and $Y = y$ are deterministic. Then, the manufacturer's optimal choice of q and $\hat{p}$ in step (i) are given by the following: if*

$$\hat{p}_{LB} > r \quad \text{or} \quad E_P\left[r - \max\{\hat{p}_{LB}, P - K_F\}\right]^+ \le E_P[r - (P + K_M)]^+, \quad (7)$$

then $q = 0$; otherwise,

$q = d/y$ *and* $\widehat{p} = \widehat{p}_{LB}$.

In the statement of Proposition 1, the first condition of (7) ensures the farmer's participation, and the second condition ensures that the contract yields lower expected profit than the case of procuring directly from the market (i.e., no contract). The proof of Proposition 1 appears in Appendix A.1.Under the optimal decision of the manufacturer given in Proposition 1, no contract is offered and signed if (7) holds, in which case the manufacturer procures all of d units from the market provided that the realised market price is favourable to do so, i.e., $P + K_M \leq$x r. If a contract is signed with the contract price $\widehat{p}_{LB} \leq r$ and the quantity $q = d/y$, then the manufacturer can procure up to d units through the contracted farmers (either though the original contracts that are ordered or through counteroffers made to these farmers). It can be shown that she procures exactly d units at price $\max\{\widehat{p}, P - K_F\}$ if $P \leq r + K_F$; otherwise she does not procure any unit. Then, her profit under this contract is $d \cdot [r - \max\{\widehat{p}_{LB}, P - K_F\}]^+$, and the manufacturer would offer this contract if the expected profit from the contract is higher than the case where no contract is offered. In summary, the manufacturer's optimal expected profit Π_M^* satisfies

.

$$\Pi_M^* = \max\left\{d \cdot E\left[r - \max\left\{\widehat{p}_{LB}, P - K_F\right\}\right]^+,\ d \cdot E[r - (P + K_M)]^+\right\}. \tag{8}$$

Since the second argument in the maximum operator above corresponds to Π_M^{NC}, it is straightforward to verify from (2) that $\Pi_M^* \geq \Pi_M^{NC}$, i.e., the manufacturer's expected profit can only increase if the contract is implemented. (This result makes sense since the manufacturer could always set the contract quantity q at zero, in which case there is no contract.) The incremental benefit of the contract is high if the farmer's opportunity cost is low (represented by low $\widehat{p}$), or the transportation costs K_F and K_M are high (resulting in a localised market of the farm crop).We note that from the choice of $\widehat{p}_{LB}$, the farmer's expected utility is always $\Pi_F^* = E[\mathcal{U}(C)]$. Thus, the gains of contract farming, if any, are captured completely by the manufacturer.

Contract without reneging

We now consider a modification of our model in which the farmer does not have any freedom to renege on the contract, and he must honour the contract price $\widehat{p}$ regardless of the outcome of the market price P. The farmer honours the contract because of legal consequences or because of the risk of severing the long-term relationship with the manufacturer. We analysed this model to address the question of whether or not eliminating the farmer's reneging behaviour would benefit the manufacturer.

When reneging is not allowed, the amount of inventory that the manufacturer receives at the end of step (i) is always $x = y \cdot q$, and each farmer receives $y \cdot \widehat{p}$ from the manufacturer. Thus, the farmer's participation constraints (5) and (6) can be modified as

$$\mathcal{U}(y \cdot \widehat{p}) \geq E_C[\mathcal{U}(C)], \quad \text{which is equivalent to} \quad \widehat{p} \geq \widehat{p}_{LB}^{NR}, \tag{9}$$

for an appropriately defined choice of $\widehat{p}_{LB}^{NR}$. (Here, the superscript NR represents "no reneging".) From (5), (6) and (9), it can easily be shown that

$$\widehat{p}_{LB} \leq \widehat{p}_{LB}^{NR}. \tag{10}$$

This result indicates that the contract price without reneging must be higher than the contract price with reneging. The difference in the contract price corresponds to the price of the farmer's option for reneging.

We can also obtain a result analogous to Proposition 1, that the optimal choice of the contract price $\widehat{p}$ is given by $\widehat{p} = \widehat{p}_{LB}^{NR}$, and that the optimal choice of q is either 0 or d/y.

More precisely,

$$\begin{cases} q = d/y \quad \text{and} \quad \widehat{p} = \widehat{p}_{LB}^{NR} & \text{if } r - \widehat{p}_{LB}^{NR} \geq E_P[r - (P + K_M)]^+ \\ q = 0 & \text{otherwise.} \end{cases}$$

From this, we can obtain that the manufacturer's expected profit is given by

$$\Pi_M^{NR} = \max\left\{d \cdot \left(r - \widehat{p}_{LB}^{NR}\right),\ d \cdot E[r - (P + K_M)]^+\right\}. \tag{11}$$

As before, we obtain from (2) that $\Pi_M^{NR} \geq \Pi_M^{NC}$. Also, we can also show that the farmer's expected utility satisfies $\Pi_F^{NR} = E[\mathcal{U}(C)]$, and the manufacturer captures all the gains of contract farming. Thus, even if reneging is not allowed, the manufacturer prefers the case with contract to the case without contract. However, the relative ranking between the two contracts (with or without reneging) is not clear, and this question is investigated below.

Comparison of the three models

Thus far, we have considered and analysed each of the three models that distinguish themselves based on the three types of contracts: no contract, contract with reneging and contract without reneging. We now compare these three models, focusing on the expected profit of the manufacturer.

We first note that in all of the three models, the expected utility of the farmer remains the same at $E[\mathcal{U}(C)]$ — this is an artifact of our modelling assumption that the manufacturer is the Stackelberg leader. We now compare the manufacturer's expected profits in these three models. From (8), (11) and (2),

$$\Pi_M^* = d \cdot \max\left\{E\left[r - \max\left\{\widehat{p}_{LB}, P - K_F\right\}\right]^+, E[r - (P + K_M)]^+\right\},$$

$$\Pi_M^{NR} = d \cdot \max\left\{\left(r - \widehat{p}_{LB}^{NR}\right),\ E[r - (P + K_M)]^+\right\}, \quad \text{and}$$

$$\Pi_M^{NC} = d \cdot E[r - (P + K_M)]^+.$$

(The above quantities Π_M^*, Π_M^{NR}, and Π_M^{NC} refer to the manufacturer's expected profit under contract with reneging, contract without reneging, and no contract, respectively.)

As discussed before, $\Pi_M^{NC} \le \min\{\Pi_M^*, \Pi_M^{NR}\}$, which we have noted before. This result makes sense since the manufacturer always has the option of refusing to offer any contract, and thus the availability of the contract should not decrease her expected profit.

Case: risk-neutral farmer

We are interested in comparing the manufacturer's expected profit under the two different types of the contracts: one with reneging and one without reneging. In general, a ranking between these contracts cannot be established (which can be deduced from the expressions of Π_M^* and Π_M^{NR} as well as (10)). Yet, in the special case that the farmer is risk-neutral, i.e., $\mathcal{U}$ is linear, such a ranking can be established. The following result shows that the manufacturer's expected profit is higher if the farmer is allowed to renege (which is our original model).

Theorem 2. *Suppose that both $D = d$ and $Y = y$ are deterministic. If the farmer is risk-neutral, then*

$$\prod_M^{NC} \le \prod_M^{NR} \le \prod_M^{*}.$$

Proof. To prove the ranking between Π_M^{NR} and Π_M^* in the above statement, note that a comparison of (5) and (9) implies that $E\max\{\widehat{p}_{LB}, P - K_F\} = \widehat{p}_{LB}^{NR}$. Then, from the expressions of Π_M^* and Π_M^{NR} above, we can deduce that $\Pi_M^* \ge \Pi_M^{NR}$.

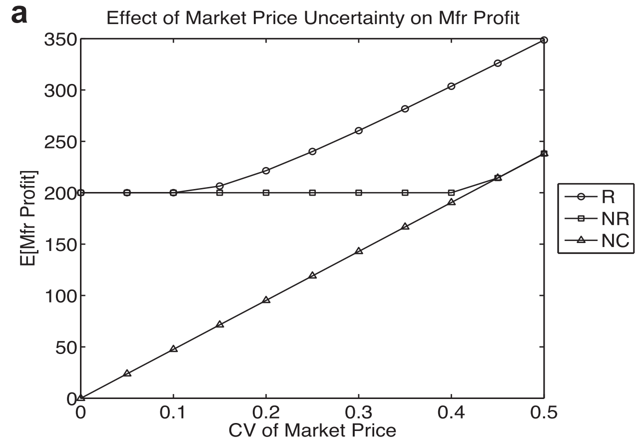
Given that reneging results in the manufacturer's uncertainty of the supply quantity and price, the phenomenon that $\Pi_M^* \ge \Pi_M^{NR}$ can be somewhat counter-intuitive. However, such an observation can be partially explained by the fact that reneging allows the manufacturer to enjoy a lower contract price since the contract price adjusts for the reneging option of the farmer (see (10)). Furthermore, reneging allows the system (consisting of both the manufacturer and farmers) the flexibility of selling the crop to the outside market when the market price P is high, and the benefit of such flexibility is captured by the manufacturer. To support this, it can be seen from (5) and (9) that the manufacturer's benefit from allowing the farmers to renege increases as P becomes more variable.

We present the results of a numerical computation. Throughout the paper, we use the following parameters unless otherwise stated:

- $r = 6$, $K_M = K_F = 0.5$, $C = 5$.
- $E[P] = 5.5$, $D = d = 200$, $Y = y = 1.0$.
- P is uniformly distributed and $CV[P] = 0.2$.

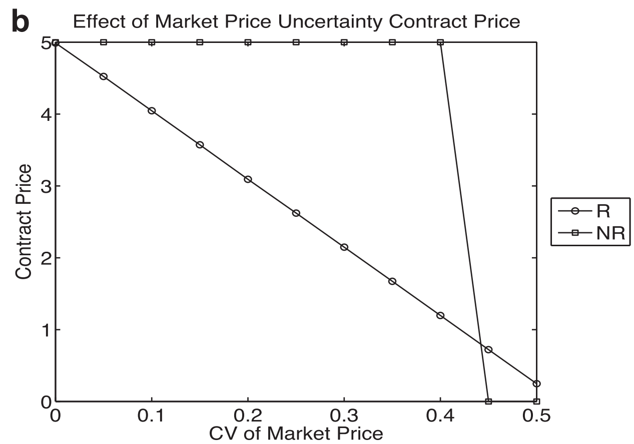
Recall that in this section we assume deterministic $D = d$ and $Y = y$. In Fig. 1, we observe the impact of the volatility in the market price P as we vary its coefficient of variation (CV). We observe that the manufacturer's profit without any contract, Π_M^{NC}, increases with the volatility in P. This result, which can in fact be deduced from (2), follows from the fact that the manufacturer has the flexibility of producing only when the market price is favourable in comparison to the sales price r. We also observe that, without reneging, the manufacturer's profit and the contract price remain constant — except when the market price uncertainty is so high that it would be beneficial to abandon the contract (which occurs when CV of P exceeds 0.4 in the plots). With reneging, the manufacturer's expected profit is also high

a



Market Price Volatility vs. Manufacturer's Expected Profits Π_M

b



Market Price Volatility vs. Contract Price $\hat{p}$

Figure 1 Effect of the volatility in market price P. Deterministic Y and D. Risk-neutral farmer. R, NR and NC refer to the contract with reneging, contract with no reneging, and no contract, respectively.

when the market price is volatile. The increase in her profit is due to the fact that she can offer a lower contract price $\widehat{p}$ to the farmer, who sees an increase in the potential value resulting from the option of reneging. The numerical results confirm the relative ranking of the manufacturer's expected profits given in Theorem 2.

Remark: For the numerical results, we have taken the approach of discretising each random variable by collecting 500 samples based on equally-spaced fractiles.

Case: risk-averse farmer

If the farmer is risk-averse, then the expected profit ranking result of Theorem 2 no longer holds. Since the farmer's revenue is deterministic when there is no reneging but it is stochastic when there is reneging, the farmer's threshold for accepting a contract price is increased with risk aversion. (Mathematically, (5) and (9) together imply that $E\max\{\widehat{p}_{LB}, P - K_F\} \geq \widehat{p}_{LB}^{NR}$ under risk aversion, and therefore it is possible that Π_M^* may be lower than Π_M^{NR}.)

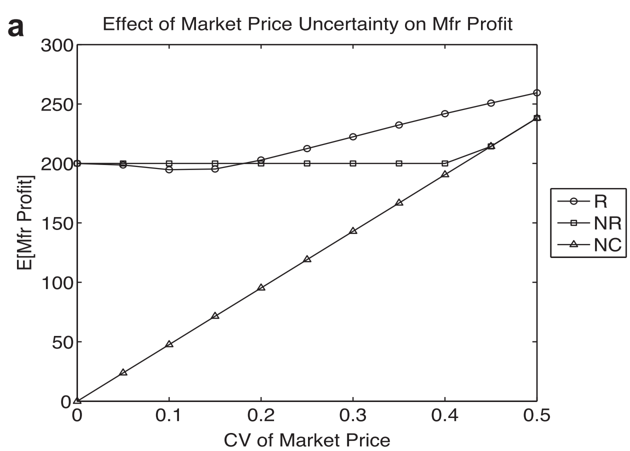
For our numerical computation, we adopt the following utility function $\mathcal{U}(z) = [1 - \exp(-\lambda z)]/\lambda$ where $\lambda > 0$. Since $\mathcal{U}(z) \to z$ as $\lambda \downarrow 0$, we also define $\mathcal{U}(z) = z$ if $\lambda = 0$. Note that this utility function displays the "constant absolute risk aversion" property, i.e., $-\mathcal{U}''(z)/\mathcal{U}'(z)$ is a constant. We make the following assumption unless otherwise stated:

- $\lambda = 0.2$ in the definition of $\mathcal{U}(z) = [1 - \exp(-\lambda z)]/\lambda$.

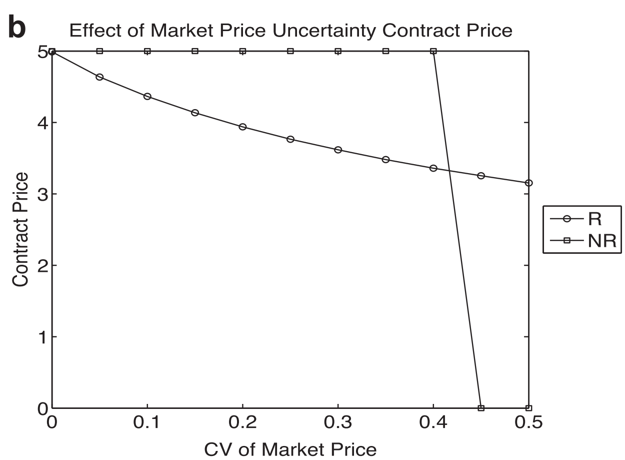
Fig. 2 plots the effect of the market price uncertainty on the manufacturer's profit and the contract price when the farmer is risk-averse. While this figure displays a similar pattern as in Fig. 1 (where the farmer is risk-neutral), we remark on how Fig. 2 differs from Fig. 1. While the contract price remains the same if the contract does not allow reneging, it is higher in the risk-neutral case if the contract allows reneging. Such an increase in the contract price can be accounted for by the farmer's aversion to profit uncertainty, which results in the decrease in the manufacturer's expected profit. Thus, the relationship $\Pi_M^{NR} \leq \Pi_M^*$ may not hold (see Fig. 2(a) when the CV of P is 0.10 or 0.15).

In Fig. 3, we investigate the impact of the farmer's risk aversion. We observe that the change in the risk aversion does not affect the manufacturer's expected profit if there is no contract or if the contract does not allow reneging. In the case where the contract allows reneging, as the risk aversion parameter λ increases, the contract price increases and the manufacturer's expected profit decreases.

In summary, we observe that contracts can significantly improve the expected profit of the manufacturer. The manufacturer prefers the contract with reneging to the contract without reneging when the farmer is risk-neutral, but this benefit decreases (and may become negative) as the farmer's risk aversion increases. Also, the reneging

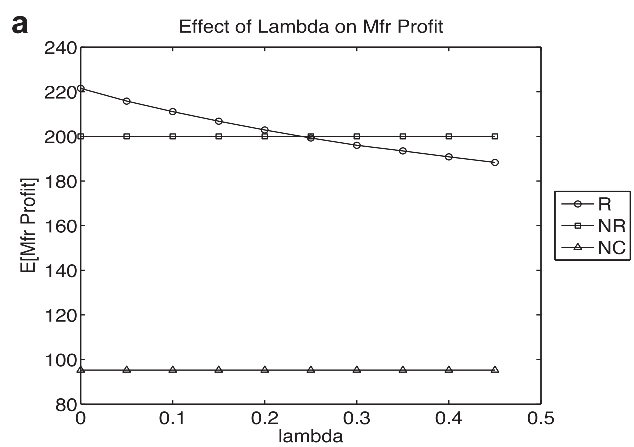


Market Price Volatility vs. Manufacturer's Expected Profits Π_M

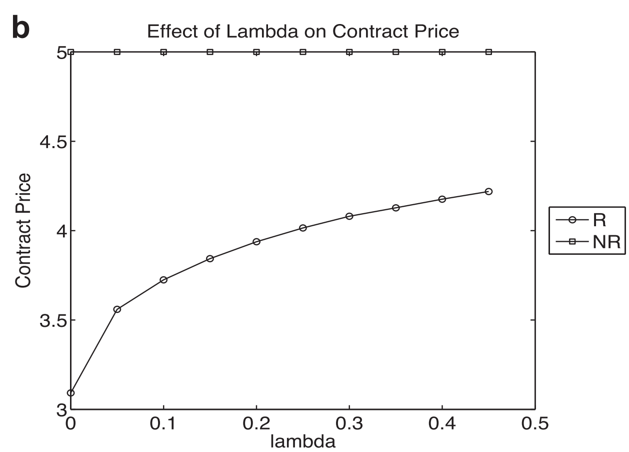


Market Price Volatility vs. Contract Price $\hat{p}$

Figure 2 Effect of the volatility in market price P. Deterministic Y and D. Risk-averse farmer with $\lambda = 0.2$. *R*, *NR* and *NC* refer to the contract with reneging, contract with no reneging, and no contract, respectively.



Farmer's Risk Aversion vs. Manufacturer's Expected Profits Π_M



Farmer's Risk Aversion vs. Contract Price $\hat{p}$

Figure 3 Effect of the farmer's risk aversion λ. Deterministic Y and D. *R*, *NR* and *NC* refer to the contract with reneging, contract with no reneging, and no contract, respectively.

feature becomes more attractive as the market price becomes more volatile.

Stochastic yield and demand

In the previous section, we have assumed that while the market price P and the opportunity cost C are stochastic, the yield Y and demand D are deterministic. This assumption made the analysis relatively simple. In this section, we allow both Y and D to be also stochastic. We follow an organisational structure similar to that of the previous section. We first study the case where there is no contract. Next, analyse the original model with reneging followed by a study of the case without reneging. Finally, we compare the manufacturer's expected profits.

A benchmark model: without contract

We first consider the benchmark case where no contract is offered when both Y and D are stochastic. In this case, the farmer's expected utility is $E_C[\mathcal{U}(C)]$. The manufacturer's expected profit is, from (1),

$$\Pi_M^{NC} = E_{P,Y}\left[\max_{z\geq 0}\{-z\cdot(P+K_M)+rE_D[\min\{D,z\}]\}\right].$$

Contract with reneging: analysis

We now address the original model described in the section entitled "Model" where the manufacturer may offer a contract that allows the farmer to renege once the market price is realised. We analyse the decisions in the sequence of events in a backward manner.

Step (iv): manufacturer's decision for z

The manufacturer makes her decision for the production quantity z in step (iv). At this point, uncertainties involving $Y = y$ and $P = p$ have already been resolved, but D is still unknown. Recall that x is the amount of farm crop that the farmers have delivered to the manufacturer according to the original terms of the contract, and that there are a total of $y\cdot q$ units of the crop harvested by all the farmers, where $x \leq y\cdot q$. Thus, it follows that the manufacturer can procure up to $y\cdot q - x$ additional units at the price of $p - K_F$ each (by making counteroffers), and any further unit at the price of $p + K_M$ each (from the market). Since the manufacturer's revenue is given by $r\cdot\min\{D, z\}$, her problem of determining optimal z in this step can be formulated as follows:

$$\max_{z\geq x} -(\min\{z,y\cdot q\}-x)\cdot(p-K_F)-[z-y\cdot q]^+\cdot(p+K_M) + r\cdot E_D[\min\{D,z\}].$$

To solve this problem, we note that it is essentially a variation of a newsvendor problem where the starting inventory is x, a limited number of units can be acquired at a lower cost $p - K_F$, and finally the remaining units can be procured at a higher cost $p + K_M$. Based on this intuition, we obtain the following proposition, which finds the solution to the above optimisation problem. We denote the optimal value of z by $z(q, p, y)$.

Proposition 3. *Suppose that both D and y are stochastic. Then, the manufacturer's optimal choice of z in step (iv) is given by*

$$z(q,p,y) = \max\{\min\{y\cdot q, z'(p)\}, z''(p), x\},$$

where $z'(p)$ and $z''(p)$ are given by

$$z'(p) = \inf\left\{z\geq 0 : P[D\leq z]\geq \frac{r-P+K_F}{r}\right\} \quad \text{and}$$
$$z''(p) = \inf\left\{z\geq 0 : P[D\leq z]\geq \frac{r-P-K_M}{r}\right\}.$$

The result of Proposition 3 can be interpreted as follows. Given the starting inventory level x, order-up-to $z'(p)$ subject to an upper bound of $y\cdot q$ — this quantity is procured by making counteroffers to reneged farmers. If the resulting inventory is still below $z''(p)$, then order-up-to $z''(p)$ by procuring from the market. The proof of Proposition 3 appears in Appendix A.2.

Step (iii): farmer's decision for reneging

The farmer's decision in step (iii) on whether to honour or renege on the contract is exactly the same as in the previous section with deterministic yield and demand. Thus, the result of equation (4) remains valid here, i.e., $x = y\cdot q$ if $p \leq \hat{p} + K_F$ and $x = 0$ otherwise. (Note that the reference to (4) should be modified accounting for the context of this section; namely, y and p should be replaced with Y and P, respectively.)

Step (i): manufacturer's decision for $\hat{p}$ and q

Moving backwards, consider the manufacturer's decision for $\hat{p}$ and q in step (i). For the contract to be acceptable to the farmers, we require a condition that ensures their participation.

$$E_{P,Y}[\mathcal{U}(Y\cdot\max\{\hat{p},P-K_F\})] \geq E_C[\mathcal{U}(C)]. \tag{12}$$

The above condition is exactly the same as (5), except that the expectation is also taken with respect to stochastic Y. Since the left-hand-side expression of the above inequality is increasing in $\hat{p}$, this condition can also be expressed as

$$\hat{p}\geq\hat{p}_{LB}$$

for some $\hat{p}_{LB}$. This inequality is similar to (6), but the exact value of $\hat{p}_{LB}$ is different from the case of deterministic yield and demand for the reason mentioned above.

We consider the impact of Y on the farmer's indifference price $\hat{p}_{LB}$. (We have already considered the impact of P in Section 3.) As the yield P increases stochastically, the left-

$$\Pi_M = E\begin{cases} -Y\cdot q\cdot\hat{p}-[z-Y\cdot q]^+\cdot(P+K_M)+r\min\{D,z\} & \text{if } P\leq\hat{p}+K_F \\ -\min\{z,Y\cdot q\}\cdot(P-K_F)-[z-Y\cdot q]^+\cdot(P+K_M)+r\min\{D,z\} & \text{otherwise.}\end{cases}$$

hand-side expression of (12) increases, making the crop more attractive, and thus the farmer is more willing to plant the crop despite the fact that the contract price may be lower, resulting in a lower value of $\widehat{p}_{LB}$. The variability of P does not have any impact on $\widehat{p}_{LB}$ under the assumptions of a risk-neutral farmer and the independence of Y and P.

The expression for the manufacturer's expected profit given in equation (1) can be simplified as follows using the farmer's reneging decision in (4):

The right-hand-side expressions in these two cases are the same except for their first terms. In attempting to unify these two cases, we introduce the following definition:

$$\vartheta(q,\widehat{p},z,p,y) = -\min\{z, y\cdot q\}\cdot\max\{\widehat{p}, p-K_F\} \\ -[z-y\cdot q]^+\cdot(p+K_M)+rE_D\min\{D,z\}.$$

(The above definition is similar to the definition of $\vartheta(q,\widehat{p},z,p)$ given in the proof of Proposition 1.) Note that if $p \le \widehat{p}+K_F$, then the choice of z must satisfy $z \ge x = p\cdot q$. Thus, it can be written that the manufacturer's expected profit satisfies

$$\Pi_M = E_{P,Y,D}\left[\vartheta(q,\widehat{p},z,P,Y)\right].$$

Then, the manufacturer's problem in step (i) is to choose $\widehat{p}$ and q to maximise

$$E_{P,Y}\left[\max_{z\ge Y\cdot q}\vartheta(q,\widehat{p},z,P,Y)\cdot 1\left[P\le\widehat{p}+K_F\right]\right] \\ +E_{P,Y}\left[\max_{z\ge 0}\vartheta(q,\widehat{p},z,P,Y)\cdot 1\left[P>\widehat{p}+K_F\right]\right], \tag{13}$$

subject to $\widehat{p} \ge \widehat{p}_{LB}$. As in our discussion in the previous section, the optimal choice of $\widehat{p}$ is $\widehat{p}_{LB}$ regardless of the value of q. Now, the following proposition shows that the above objective function is concave in q. The proof of this result is based on the preservation of concavity under the maximum and expectation operators, and appears in Appendix A.3.

Proposition 4. *Suppose that both D and Y are stochastic.*

(i) *If* $p \le \widehat{p}_{LB}+K_F$, *then* $\vartheta(q,\widehat{p}_{LB},z,p,y)$ *is jointly concave in* $\{(q,z)|q\ge 0, z\ge y\cdot q\}$.
(ii) *If* $p > \widehat{p}_{LB}+K_F$, *then* $\vartheta(q,\widehat{p}_{LB},z,p,y)$ *is jointly concave in* $\{(q,z)|q\ge 0, z\ge 0\}$.
(iii) *In the manufacturer's problem of maximizing (13) with respect to* $(\widehat{p},q)$, *the optimal choice of* $\widehat{p}$ *is* $\widehat{p}_{LB}$, *and this objective function is concave in q.*

Thus, the manufacturer's optimal expected profit, denoted by Π_M^*, is easy to find by maximizing a single-variable concave function.

Contract with reneging

We consider a modification of the original model in which the farmer now cannot renege on the contract. This is the same modification that was introduced in the previous section. The contracted farmer must sell all of his crops to the manufacturer (and the manufacturer must purchase these crops) at the contract price $\widehat{p}$.

In this case, the farmer's reneging decision in step (iii) of the model does not exist. When the manufacturer sets the contract price $\widehat{p}$ in step (i), she must consider the farmer's outside option to ensure his participation. Thus, the following constraint is imposed on the choice of $\widehat{p}$:

$$E_{P,Y}\left[\mathcal{U}(Y\cdot\widehat{p})\right]\ge E_C[\mathcal{U}(C)], \quad \text{which is equivalent to} \quad \widehat{p}\ge\widehat{p}_{LB}^{NR}, \tag{14}$$

for an appropriately defined choice of $\widehat{p}_{LB}^{NR}$. (Note that this constraint is a modification of (9) that accounts for the randomness in Y, and the value of $\widehat{p}_{LB}^{NR}$ is in general different from the quantity defined in Section 3.3.) From (12) and (14), it can be also be shown that

$$\widehat{p}_{LB}\le\widehat{p}_{LB}^{NR}.$$

In determining the optimal manufacturer's decision $(\widehat{p},q)$, it can be shown using previous arguments that her profit is non-increasing in $\widehat{p}$. Thus, $\widehat{p}=\widehat{p}_{LB}^{NR}$ is the optimal contract price if any contract is offered. If $\widehat{p}_{LB}^{NR}>r$, then the manufacturer does not make any profit from the contracted crops, and thus she does not offer any contract. Thus, each farmer's expected utility is $E_C[U(C)]$. We proceed by assuming $\widehat{p}_{LB}^{NR} \le r$. Then, from the above discussion and (1), the manufacturer's expected profit can be written as

$$E\left[\Pi_M^{NR}(q)\right]=E_{P,Y}\Big[-Y\cdot q\cdot\widehat{p}_{LB}^{NR}+\max_{z\ge Y\cdot q}\{-[z-Y\cdot q]\cdot(P+K_M) \\ +rE_D[\min\{D,z\}]\}\Big]. \tag{15}$$

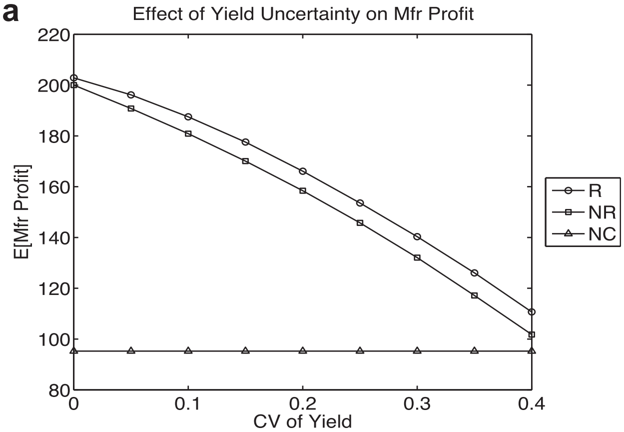
The manufacturer's objective in step (i) is to choose q to maximise the expected value $E[\Pi_M^{NR}(q)]$ where the expectation is taken with respect to Y and P as well as D; the recourse decision z is made after Y and P are realised, but before D is realised. Note that if Y is deterministic and the second purchase opportunity (represented by z) does not exist, then the above function is the objective of a newsvendor problem. Thus, our problem is an extension of the newsvendor model with the features of the random yield (see, for example, Yano and Lee (1995)) and also the second purchase opportunity with price uncertainty.

Let $z(q, Y, P)$ denote the optimal value of z in the definition (15) of $E[\Pi_M^{NR}(q)]$ for given q, Y and P. Let $z'(P)$ be defined as in the statement of Proposition 3. Clearly, $z'(P)$ is a decreasing function of P. The proof of Proposition 5 is straightforward and appears in Appendix A.3.

Proposition 5. *Consider a modification of the model where reneging is not allowed. Suppose that both D and Y are stochastic. Then,* $z(q, y, p)$ *follows an order-up-to policy with the base stock level of* $z'(p)$, *and* $E[\Pi_M^{NR}(q)]$ *is concave in q.*

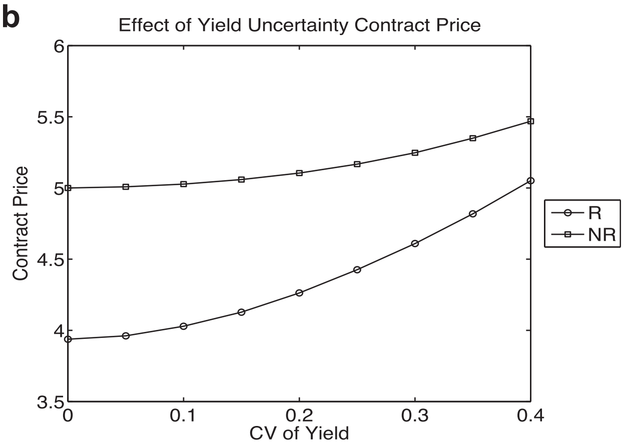
We denote the optimal expected revenue of the manufacturer by $\Pi_M^{NR}=\max_{q\ge 0}E[\Pi_M^{NR}(q)]$.

a



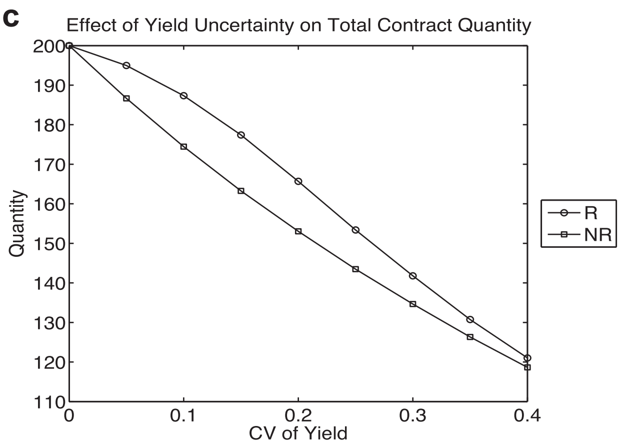
Yield Uncertainty vs. Manufacturer's Expected Profits Π_M

b



Yield Uncertainty vs. Contract Price $\hat{p}$

c



Yield Uncertainty vs. Contract Price $\hat{p}$

Figure 4 Effect of the yield uncertainty. Deterministic *D*. Risk-averse farmer with $\lambda = 0.2$. *R*, *NR* and *NC* refer to the contract with reneging, contract with no reneging, and no contract, respectively.

Comparison of the three models

For the stochastic yield and demand, we have considered each of the three cases distinguished by the types of the contract, and described how to solve each of the cases. In this section, we use numerical results to compare these three cases. Our example is based on the one introduced previously.

Effect of yield uncertainty

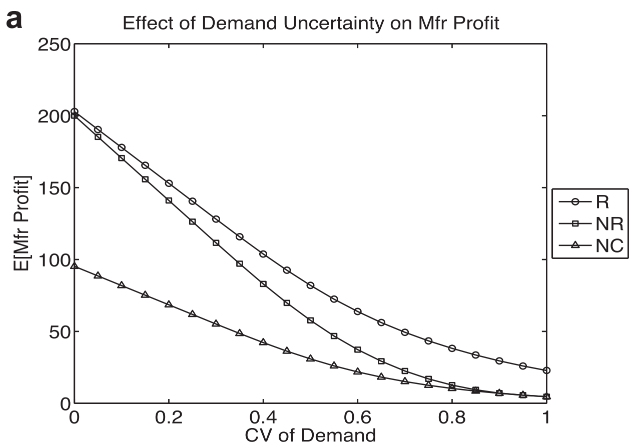
We first study the impact of yield uncertainty by using a uniform distribution for the yield distribution. In Fig. 4, we fix the mean of the yield distribution $E[Y] = 1$, and vary the coefficient of variation (CV) of the yield distribution Y between 0 and 0.4. From Fig. 4(b), we observe that the contract price is increasing as the yield uncertainty increases, regardless of whether reneging is allowed or not. If reneging is not allowed, the uncertainty in yield produces the farmer's revenue uncertainty, which increases the contract price since the farmer is risk-averse. If reneging is allowed, the compounding uncertainty of yield and price causes a steeper increase in the contract price.

In Fig. 4(c), we plot the contract quantity q as a function of the yield uncertainty. As yield becomes more variable, the manufacturer is less willing to commit a large quantity on the contract due to the fear of disposing excess farm crop that she does not use; she is more inclined to use a more reliable option of purchasing from the outside market. Also note that the contract quantity is always higher with reneging compared to the case without reneging. This makes sense since reneging implies that the manufacturer does not always receive the contract quantity. When the market price is low, farmers do not renege and the manufacturer receives all of the produced farm crop. This is a desirable outcome for the manufacturer since the margin is high and the manufacturer would like to produce a large quantity of the product. When the market price is high, farmers renege on the contract, and this is also the desired outcome for the manufacturer since the manufacturer does not want to produce a large quantity when the margin is low. Thus, under the possibility of reneging, the contract quantity is determined by considering a conditional distribution of the price that it is below a certain threshold (thus resulting in a higher production quantity).

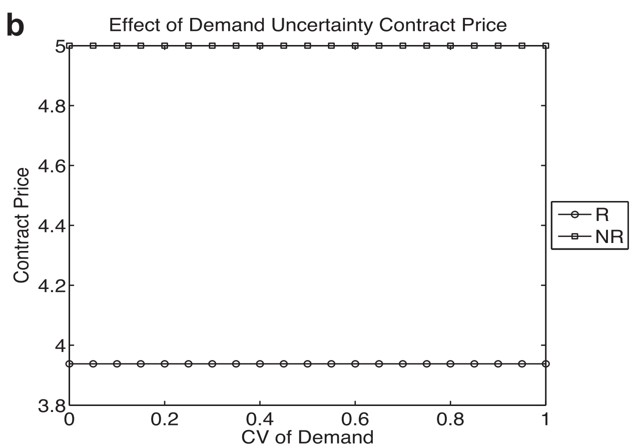
The above discussion leads to the following observation of the profit ranking $\Pi_M^{NC} \leq \Pi_M^{NR} \leq \Pi_M^*$ found in Fig. 4(a). We also observe that the expected profit decreases in yield uncertainty, which can be explained by the fact that the manufacturer makes decisions before uncertainties are resolved, and that she has to pay a higher contract price to the farmer. Fig. 4 shows that the reduction in the yield uncertainty produces an increased profit to the manufacturer, an increased farmer's willingness for contract farming, and an increase in use of the contract. In practice, the yield uncertainty reduction can be achieved through the availability of technology, irrigation and fertilizer.

Effect of demand uncertainty

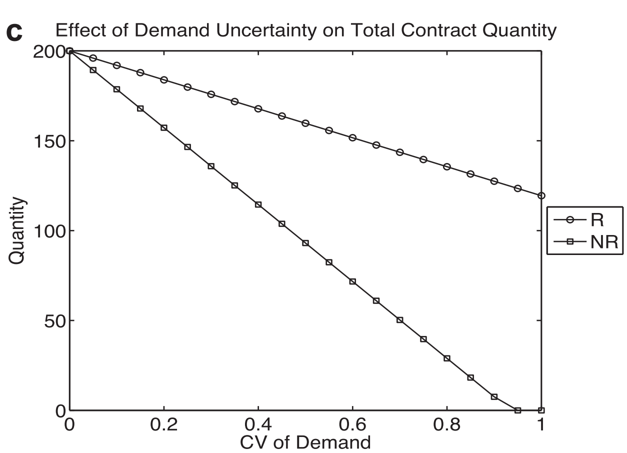
In Fig. 5, we study the impact of demand uncertainty. We use a normal distribution for demand D where the mean is fixed at $E[D] = 200$ and the coefficient of variation (CV)



Demand Uncertainty vs. Manufacturer's Expected Profits Π_M



Demand Uncertainty vs. Contract Price $\hat{p}$



Demand Uncertainty vs. Contract Price $\hat{p}$

Figure 5 Effect of the yield uncertainty. Deterministic *D*. Risk-averse farmer with $\lambda = 0.2$. *R*, *NR* and *NC* refer to the contract with reneging, contract with no reneging, and no contract, respectively.

varies between 0 and 0.4. Any negative demand is truncated at 0. (The yield is deterministic at $Y = 1$.)

Fig. 5(b) shows that the contract price does not depend on demand uncertainty. This can be deduced directly from the expressions of the contract prices $\widehat{p}_{LB}^{NR}$ and $\widehat{p}_{LB}^{NR}$ given in (14) and (12). In Fig. 5(c), we observe that the contract quantities decline in demand uncertainty, and that the contract quantity is higher if reneging is allowed. Both of these observations can be explained by a similar argument used in the yield uncertainty case.

From Fig. 5(a), the manufacturer's profits decline in demand uncertainty in all three cases. We see that the benefit of the contract is highest when the demand is deterministic, and this benefit decreases in demand uncertainty. While this benefit erodes to zero if the contract does not allow reneging, it is still possible to squeeze some benefit with the contract that does allow for reneging. In fact, the benefit of reneging (the difference of the expected profits under two types of contract) actually increases in demand uncertainty.

From the numerical results, we conclude that putting in place a contract provides an increased expected profit to the manufacturer compared to the case without any contract, and also that the reneging feature can be beneficial to the manufacturer. In summary, reneging is particularly useful when market price is volatile, the farmer is risk-neutral, and the demand is uncertain.

Concluding remarks

In this paper, we have considered the farmer's reneging option in the context of contract farming, and demonstrated that such an option can improve the expected profit of the manufacturer. Such an improvement can be expected when the farmer is risk-neutral (such that some fluctuation in his profit would not be undesirable), when the market price is volatile (such that the manufacturer's production quantity can be more freely adjusted to the market price), and when the demand uncertainty is high (such that the profit function is not very sensitive to supply reduction).

We expect that the contract farming framework outlined in this paper would serve as a prototype as we develop a more detailed high-accuracy contract farming model that could be used for multinational corporations engaged in the practice of contract farming. This involves identifying the farmer's decision criteria, as modeled in Huh and Lall (2008), and understanding the manufacturer's multi-faceted objective structure — as well as setting up appropriate oversight and incentive mechanisms by regulatory agencies.

A specific extension of the work presented here would be to consider covariation in the uncertainty distribution of yield *Y* and market price *P*. This is particularly relevant in situations where aggregate regional production is highly sensitive to regional climate exigencies and regional market prices are affected by regional production given transportation costs and constraints. Under such conditions, public or private investment in reducing yield uncertainty through (i) investment in irrigation technology; (ii) development and communication of weather forecasts; and (iii) the use of appropriate seeds and fertilizers, would lead to a reduction in price uncertainty and possibly a decoupling of the price and yield distributions. The framework presented in this paper readily lends itself to an analysis of how much investment the contract offerer, the farmer, or even the government may want to make to reduce yield uncertainty through the means indicated above, or to mitigate its impact by purchasing appropriately priced insurance products that address price and/or yield uncertainty. We intend to pursue these investigations using specific settings in India to provide the applied context.

An additional goal is to generalise our analysis to a framework in which farmers may have heterogeneous utility functions and opportunity costs. Unfortunately, much of the straightforward analysis that is carried out in this paper would not readily extend to an asymmetric economic environment. To complicate matters even further, such information may be private. If it is, then the task of designing a procedure to truthfully elicit it from the farmers would raise nontrivial theoretical questions. We plan to address these issues in future work.

Acknowledgment

This research was a part of a PepsiCo Foundation funded project entitled *Improving Rural Water and Livelihood Outcomes in India, China, Africa and Brazil*. The authors thank Ingrid Cheh, an undergraduate student at Columbia University, for her assistance. We would also thank the referee board for their helpful comments.

Appendix A
Proofs

A.1 Proof of Proposition 1

Proof. If $\widehat{p}_{LB} > r$, then any contract price that would be acceptable to the farmer results in loss for the manufacturer. Since there would be no contract that would be acceptable to both the manufacturer and farmers in this case, we set $q = 0$. We proceed by assuming otherwise.

The manufacturer's problem is to select q and $\widehat{p}$ to maximise her expected profit, where expectation is taken over P, given that the optimal recourse decision z in (3) is made. (Recall (1) for the expression of her profit.) Define

$$\vartheta(q,\widehat{p},z,p) = -\min\{z, y\cdot q\}\cdot\max\{\widehat{p}, p-K_F\} - [z-y\cdot q]^+\cdot(p+K_M) + r\min\{d,z\},$$

which represents the net profit of the manufacturer associated with the initial decisions $(q,\widehat{p})$, the market crop price p and the recourse action z. Then, from the expression of x given by (4), it can be shown that the manufacturer's expected profit for a given pair of q and $\widehat{p}$ is

$$E_P\left[\max_{z\geq y\cdot q}\vartheta(q,\widehat{p},z,P)\cdot 1\left[P\leq\widehat{p}+K_F\right]\right] + E_P\left[\times\max_{z\geq 0}\vartheta(q,\widehat{p},z,P)\cdot 1\left[P>\widehat{p}+K_F\right]\right],$$

subject to the constraint (5) which ensures that the farmer would accept the contract. (To prove this assertion, we use the fact that $x = y\cdot q$ in case of $P\leq\widehat{p}+K_F$, and $x = 0$ in case of $P>\widehat{p}+K_F$.) Note that the constraint (5) is equivalent to (6), and that the two optimisation problems in the above expression are identical except for the range of z for the maximum operator. Since the first maximisation problem is more restrictive and $\vartheta(q,\widehat{p},z,p)$ is increasing in $\widehat{p}$, it follows that $\widehat{p}$ should be set at $\widehat{p}_{LB}$ regardless of the value of q.

Now we investigate the optimal choice of q. Since the demand and the yield are deterministically d and y respectively, there is no reason to set q higher than d/y. In fact, the contract quantity q should be one of the extreme solutions, i.e., $q\in\{0, d/y\}$. Moreover, it can be shown that, if $q = d/y$, then the manufacturer's profit is $E_P[r-\max\{\widehat{p}_{LB}, P-K_F\}]^+$ multiplied by d; if $q = 0$, then it is $E_P[r-(P+K_M)]^+$ multiplied by d. By comparing these two quantities, we obtain $q = d/y$ or $q = 0$ accordingly.

A.2 Proof of Proposition 3

Proof. Since $x\cdot(p - K_F)$ does not depend on z, the manufacturer's objective is equivalent to maximising $R(z)$ over $z\geq x$, where we define

$$\mathcal{R}(z) = -z\cdot(p-K_F) - [z-y\cdot q]^+\cdot(K_F+K_M) + r\cdot E_D[\min\{D,z\}].$$

Note that R is concave in z; in fact, it represents a newsvendor revenue function where the purchase cost is convex and piecewise linear in z. We separately consider optimising R over two intervals: $[0,\ y\cdot q]$ and $[y\cdot q,\ \infty]$.

In the first interval $[0,\ y\cdot q]$, we have

$$\mathcal{R}(z) = -z\cdot(p-K_F) + r\cdot E_D[\min\{D,z\}].$$

Thus, the optimal value of z in this interval is $\min\{y\cdot q,\ z'(p)\}$, where $z'(p)$ is the solution to the newsvendor problem with the overage cost of $p - K_F$ and the underage cost of $r - p + K_F$.

In the second interval $[y\cdot q,\ \infty]$,

$$\mathcal{R}(z) = y\cdot q\cdot(K_F+K_M) - z\cdot(p+K_M) + r\cdot E_D[\min\{D,z\}].$$

Note that the first term in the above expression does not depend on z. The optimal value of z in this interval is $\max\{y\cdot q,\ z''(p)\}$, where $z''(p)$ is the solution to the newsvendor problem with the overage cost of $p + K_M$ and the underage cost of $r - p - K_M$.

From the definitions of $z'(p)$ and $z''(p)$, it follows that $z''(p)\leq z'(p)$. The above analysis shows that the value of z maximising R over the interval $[0,\ \infty]$ is

$$\begin{cases} z''(p) & \text{if } y\cdot q\leq z''(p) \\ y\cdot q & \text{if } y\cdot q\in[z''(p), z'(p)] \\ z'(p) & \text{if } z'(p)\leq y\cdot q\,, \end{cases} \quad \text{and}$$

which can be written as $\max\{\min\{y\cdot q,\ z'(p)\},\ z''(p)\}$. Therefore, by the convexity of R, the value of z maximising R in the interval $[x,\ \infty]$ is $\max\{\min\{y\cdot q,\ z'(p)\},\ z''(p),\ x\}$.

A.3 Proof of Proposition 4

Proof.

(i) Suppose $p \leq \widehat{p} + K_F$ and $z \geq y \cdot q$. Then, from the definition of $\vartheta(q, \widehat{p}, z, p, y)$,

$$\vartheta(q, \widehat{p}, z, p, y) = -y \cdot q \cdot \widehat{p} - [z - y \cdot q] \cdot (p + K_M) + rE_D \min\{D, z\}.$$

Since the first two terms are linear in (q, z) and the last term is independent of q and concave in z, it follows that the above expression is also in (q, z).

(ii) Suppose $p > \widehat{p} + K_F$. Then, since $\min\{z, y \cdot q\} + [z - y \cdot q]^+ = z$, we have

$$\begin{aligned}\vartheta(q, \widehat{p}, z, p, y) = & -\min\{z, y \cdot q\} \cdot (p - K_F) \\ & -[z - y \cdot q]^+ \cdot (p + K_M) + rE_D \min\{D, z\} \\ = & -z \cdot (p - K_F) - [z - y \cdot q]^+ \cdot (K_F + K_M) \\ & + rE_D \min\{D, z\},\end{aligned}$$

which is clearly concave in (q, z).

(iii) The optimality of $\widehat{p} = \widehat{p}_{LB}$ follows the argument similar to the one used in the section "Deterministic yield and demand". The concavity of (13) with respect to q for fixed $\widehat{p}$ follows from parts (i) and (ii) above.

Proof of Proposition 5

Proof. We first consider the second replenishment decision z given the earlier decision q and the realisations of y and p. The maximisation problem in (15) is concave in z. Let $z'(p)$ be the optimal value of z provided that $q = 0$, and it is easy to show that

$$z'(p) = \inf\left\{z \geq 0 : P[D \leq z] \geq \frac{r - p + K_F}{r}\right\}.$$

Then, the optimal choice of z follows the order-up-to-$z'(p)$ policy, i.e., $\max\{y \cdot q, z'(p)\}$.

Now, the concavity of $E[\Pi_M^{NR}(q)]$ follows from the joint concavity of $-[z - y \cdot q] \cdot (p + K_M) + rE_D[\min\{D, z\}]$ with respect to (q, z) in (15) and the preservation of the concavity under the maximum operator.

References

Allen, S. J., & Schuster, E. W. (2004). Controlling the risk for an agricultural harvest. *Manufacturing & Service Operations Management, 6*(3), 225–236.

Brennan, D. (2004). Public policy issues in supply-chain management: proceedings of a workshop held in Bali, Indonesia, 19–22 August 2003. In G. I. Johnson, & P. J. Hofman (Eds.), *Agriproduct supply-chain management in developing countries* (pp. 152–163). Australian Centre for International Agricultural Research.

Burer, S., Jones, P. C., & Lowe, T. J. (2008). Coordinating the supply chain in the agricultural seed industry. *European Journal of Operational Research, 185*(1), 354–377.

Delpierre, M. (2005). *Farm input supply under contractual insecurity*. Working Paper.

Domberger, S. (1998). *The contracting organization: A strategic guide to outsourcing*. Oxford University Press.

Food and Agriculture Organization. (2001). *Contract farming: Partnerships for growth*. Technical report. Food and Agriculture Organization of the United Nations.

Food and Agriculture Organization. (2002). *World agriculture: Towards 2015/2030*. Technical report. Food and Agriculture Organization of the United Nations.

Ghosh, C., & Raychaudhuri, A. (2010). *Model of contract farming and price risk with special reference to Indian agriculture*. Working Paper.

Glen, J. J. (1987). Mathematical models in farm planning: a survey. *Operations Research, 35*(5), 641–666.

He, Y., & Zhang, J. (2008). Random yield risk sharing in a two-level supply chain. *International Journal of Production Economics, 112*(2), 769–781.

Huh, W. T., & Lall, U. (2008). *Optimizing crop choice and irrigation allocation under contract farming*. Technical report. Columbia University.

Jones, P. C., Kegler, G., Lowe, T. J., & Traub, R. D. (2003). Managing the seed-corn supply chain at Syngenta. *Interfaces, 33*(1), 80–90.

Jones, P. C., Lowe, T. J., & Traub, R. D. (2002). Matching supply and demand: the value of a second chance in producing seed corn matching supply and demand: the value of a second chance in producing seed corn. *Review of Agricultural Economics, 24*(1), 222–238.

Jones, P. C., Lowe, T. J., Traub, R. D., & Kegler, G. (2001). Matching supply and demand: the value of a second chance in producing hybrid seed corn. *Manufacturing & Service Operations Management, 3*(2), 122–137.

Kazaz, B. (2008). *Pricing and production planning under supply uncertainty*. Technical report. Syracuse University.

Key, N., & Runsten, D. (1999). Contract farming, smallholder, and rural development in Latin America: the organization of agro-processing firms and the scale of outgrower production. *World Development, 27*(2), 381–401.

Khairnar, S., & Yeleti, V. (2005). Contract farming in India: impact and Implications. In R. Chand, & M. S. Swaminathan (Eds.), *India's agricultural challenges: Reflections on policy, technology and other issues* (pp. 105–128). Centad Centre for Trade and Development.

Li, W., Li, W., & Li, Z. (2004). Irrigation and fertilizer effects on water use and yield of Spring wheat in semi-arid regions. *Agricultural Water Management, 67*(1), 35–46.

Lowe, T. J., & Preckel, P. V. (2004). Decision technologies for agribusiness problems: a brief review of selected literature and a call for research. *Manufacturing & Service Operations Management, 6*(3), 201–208.

Nayak, A. K. (2000). *Optimizing asymmetries for sustainability: A prism for agricultural research for development*. Working paper.

Patlolla, S. (2010). *Managing quantity, quality and timing in cane sugar production: Ex post marketing permits or ex ante production contracts?*. Working paper.

Plambeck, E. L., & Taylor, T. A. (2005). Sell the plant? The impact of contract manufacturing on innovation, capacity and profitability. *Management Science, 51*(1), 133–150.

Plambeck, E. L., & Taylor, T. A. (2006). Partnership in a dynamic production system with unobservable actions and non-contractible output. *Management Science, 52*(10), 1509–1527.

Plambeck, E. L., & Taylor, T. A. (2007a). Implications of breach remedy and renegotiation design for innovation and capacity. *Management Science, 53*(12), 1859–1871.

Plambeck, E. L., & Taylor, T. A. (2007b). Implications of renegotiation for optimal contract flexibility and investment. *Management Science, 53*(12), 1872–1886.

Plambeck, E. L., & Taylor, T. A. (2007c). Simple relational contracts to motivate capacity investment: price only vs. price and quantity. *Manufacturing & Service Operations Management, 9*(1), 94—113.

Plambeck, E. L., & Taylor, T. A. (2007d). Supply chain relationships and contracts: the impact of repeated interaction on capacity investment and procurement. *Management Science, 53*(10), 1577—1593.

Reardon, T., & Barrett, C. B. (2000). Agroindustrialization, globalization, and international development: an overview of issues, patterns, and determinants. *Agricultural Economics, 23*, 195—205.

Reardon, T., & Berdegue, J. A. (2002). The rapid rise of supermarkets in Latin America: challenges and opportunities for development. *Development Policy Review, 20*(4), 371—388.

Rehber, E. (1998). *Vertical integration in agriculture and contract farming*. Technical Report 46. Food Marketing Policy Center, University of Connecticut.

Runsten, D., & Key, N. (1996). *Contract farming in developing countries: Theoretical aspects and analysis of some Mexican cases*. Technical report. Santiago, Chile: United Nations Economic Commission for Latin America and the Caribbean.

Salas, P. C. (2009). *The role of market power in agricultural contracts*. Working paper.

Sivramkrishna, S., & Jyotishi, A. (2008). Monopsonistic exploitation in contract farming: articulating a strategy for grower cooperation. *Journal of International Development, 20*(3), 280—296.

Swinnen, J. F. M., & Vandeplas, A. (2007). *Contracting, competition, and rent distribution theory and empirical evidence from developing and transition countries*. Technical report. Katholieke Universiteit Leuven.

Thome, K. E., & Sexton, R. J. (2007). *Contracts, quality, and default: Endogenizing a buyers rejection rate*. Working Paper.

Ülkü, S., Toktay, L. B., & Yücesan, E. (2007). Risk ownership in contract manufacturing. *Manufacturing & Service Operations Management, 9*(3), 225—241.

UN Millennium Project. (2005). *Investing in development: A practical plan to achieve the millennium goals*. New York.

Warning, M., & Key, N. (2002). The social performance and distributional consequences of contract farming: an equilibrium analysis of the Arachide de Bouche Program in Senegal. *World Development, 30*(2), 255—263.

Yano, C. A., & Lee, H. L. (1995). Lot sizing with random yields: a review. *Operations Research, 43*(2), 311—334.