

Climate elasticity of streamflow in the United States

A. Sankarasubramanian and Richard M. Vogel

Department of Civil and Environmental Engineering, Tufts University, Medford, Massachusetts

James F. Limbrunner

Haley and Aldrich, Inc., Manchester, New Hampshire

Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.

Abstract. Precipitation elasticity of streamflow, ε_P , provides a measure of the sensitivity of streamflow to changes in rainfall. Watershed model-based estimates of ε_P are shown to be highly sensitive to model structure and calibration error. A Monte Carlo experiment compares a nonparametric estimator of ε_P with various watershed model-based approaches. The nonparametric estimator is found to have low bias and is as robust as or more robust than alternate model-based approaches. The nonparametric estimator is used to construct a map of ε_P for the United States. Comparisons with 10 detailed climate change studies reveal that the contour map of ε_P introduced here provides a validation metric for past and future climate change investigations in the United States. Further investigations reveal that ε_P tends to be low for basins with significant snow accumulation and for basins whose moisture and energy inputs are seasonally in phase with one another. The Budyko hypothesis can only explain variations in ε_P for very humid basins.

1. Introduction

Hundreds, possibly thousands, of studies are now available which document the sensitivity of streamflow to climate for river basins all over the world. Most hydrologic climate sensitivity studies involve calibrating a conceptual deterministic watershed model, and then varying the model's atmospheric inputs, to observe the resulting changes in streamflow. Schaake [1990], Nash and Gleick [1991], and Jeton *et al.* [1996] have performed this type of study. Another approach is to analytically derive the sensitivity of streamflow in terms of model parameters [Schaake, 1990]. A third approach is to fit multivariate regional hydrologic models using climate and streamflow data for many basins in a region [Vogel *et al.*, 1999]. A fourth approach is to empirically estimate changes in streamflow which resulted from historical changes in climate [Risbey and Entekhabi, 1996]. A fifth approach is to use multivariate statistical methods to estimate the relationship between climate and streamflow at a single site [Revelle and Waggoner, 1983]. Of all these approaches the use of conceptual deterministic watershed models is by far the most common because such models are able to model the complex spatial and temporal variations in evapotranspiration, soil moisture, groundwater, and streamflow. Leavesley [1994] provides a more detailed discussion of the advantages of conceptual watershed models for modeling climate change impacts.

In spite of the advantages of using conceptual watershed models in climate change studies their validation still remains a fundamental challenge. Climate sensitivity analyses performed on the same basin using different conceptual watershed models can lead to significantly different results. Worse yet, climate sensitivity analyses performed on the same basin using identical conceptual watershed models can lead to remarkably different results. For example, Nash and Gleick

[1991] and Schaake [1990] used the National Weather Service River Forecasting System (NWSRFS) to perform climate sensitivity analyses on the Animas River at Durango, Colorado. When precipitation was increased by 10%, holding temperature and potential evapotranspiration constant, Nash and Gleick [1991] and Schaake [1990] reported an 11% and 20% increase, respectively, in annual streamflow. These are rather remarkable differences, especially considering that the same model was applied to the same basin in both instances. These different sensitivities are likely due to differences in model calibrations leading to differences in model parameter estimates which result in differences in the models' sensitivity to climate variations.

Vogel *et al.* [1999] use a regional multivariate regression model to document that a 10% increase in precipitation should lead, on average, to a 19% increase in annual streamflow for the entire upper Colorado River system, of which the Animas River is only a small subbasin. The regional models developed by Vogel *et al.* [1999] are based on annual climate and streamflow data from 44 basins in the upper Colorado River basin. Treating the entire upper Colorado River as a single basin, Revelle and Waggoner [1983] report an at-site multivariate regression relation between streamflow, precipitation, and temperature which documents that a 10% increase in annual precipitation leads to an 11% increase in streamflow. Who does one believe? The results of these four different estimates of the sensitivity of streamflow to precipitation for the Animas basin are summarized in Table 1. The agreement between Vogel *et al.* [1999] and Schaake [1990] is excellent, and the agreement between Nash and Gleick [1991] and Revelle and Waggoner [1983] is also quite good. However, they cannot all be correct because the two sets of studies lead to very different conclusions.

We are left with the uncomfortable feeling that definitive estimates of the sensitivity of streamflow to climate are still unavailable, or if they do exist, it is difficult to judge which investigation is plausible and which is not. The primary goal of this study is to develop a uniform, defensible, and reproducible

Copyright 2001 by the American Geophysical Union.

Paper number 2000WR900330.
0043-1397/01/2000WR900330\$09.00

Table 1. Comparison of Estimates for the Animas River Basin in Colorado

Study	Model	Percentage Increase in Annual Streamflow Resulting From a 10% Increase in Annual Precipitation
<i>Nash and Gleick</i> [1991]	NWSRFS	10.9
<i>Revelle and Waggoner</i> [1983]	at-site multivariate regression	10.5
<i>Schaake</i> [1990]	NWSRFS	19.7
<i>Vogel et al.</i> [1999]	regional multivariate regression	19.0

approach for evaluating the sensitivity of streamflow to climate. The approach must be robust, meaning it should yield similar results for a wide range of assumed hydrologic model structures. The approach must also be unbiased so that on average, over many applications, one may discern the true underlying sensitivity of streamflow to climate. This is challenging for a number of reasons. The sensitivity of streamflow to climate is itself a dynamic quantity which may change as climate changes. Furthermore, it is challenging to develop an estimator which can produce unbiased estimates of the sensitivity of streamflow to climate under different model assumptions because the development of an unbiased estimator itself has to be based on a model assumption. Our goal is to develop a robust and approximately unbiased estimator of the sensitivity of streamflow to climate which can perform well under different model assumptions. This estimator is then used for (1) constructing regional maps of the sensitivity of streamflow to climate for the continental United States, (2) evaluating and comparing our results with other climate sensitivity studies, and (3) understanding the physical processes which dominate the sensitivity of streamflow to climate. We also explore the use of climate elasticity of streamflow as an external validation statistic which may prove useful in future climate change investigations.

2. Description of Databases

Contour maps of the climate elasticity of streamflow are developed in this study using a nationwide hydroclimatologic database consisting of annual time series of streamflow Q , precipitation P , and potential evapotranspiration PE for 1291 basins in the United States. Our analyses employ the following databases.

2.1. Streamflow Database

Streamflow data are obtained from the Hydro-Climatic Data Network (HCDN), available from the U.S. Geological Survey and compiled by *Slack et al.* [1993]. This data set consists of records of average daily, monthly, and annual streamflow at 1553 sites located in the United States. Since the details of this data set are discussed by *Slack et al.* [1993] and others, we do not repeat those discussions here. This study used the annual flow records over the period 1951–1988 with at least 20 years of record and with basin areas in excess of 129 km² (50 mi²). This resulted in a total of 1291 gaged watersheds across the continental United States. Their locations are shown in Figure 1

along with each of the 18 major water resource regions defined by the U.S. Water Resources Council in 1970.

2.2. Precipitation and Potential Evaporation Data

Monthly time series of precipitation and average maximum and minimum daily temperatures were obtained for the 1291 sites from 0.5° time series grids based on the Precipitation-Elevation Regressions on Independent Slopes Model (PRISM) climate interpolation modeling system [*Daly et al.*, 1994]. PRISM uses a precipitation-elevation regression relationship to distribute point measurements to evenly spaced grid cells. PRISM is considered an improvement over other spatial interpolation methods such as inverse distance weighting or kriging because it attempts to account for orographic effects by using precipitation-elevation regression functions. *Daly et al.* [1994] found PRISM estimates of precipitation reproduced expected patterns over areas with complex topography, such as mountainous regions.

The monthly climate time series grids were spatially averaged over each HCDN basin using a geographic information system. To accomplish this task, the watershed boundaries of the 1291 river basins were outlined using a digital elevation map of the United States. The end result is a unique national time series data set of monthly precipitation and temperature measurements for the HCDN basins over the period 1951–1988. Estimates of monthly potential evaporation were obtained using a method introduced by *Hargreaves and Samani* [1982] which is based on monthly time series of average minimum and maximum temperature data along with extraterrestrial solar radiation. Extraterrestrial solar radiation was estimated for each HCDN basin by computing the solar radiation over 0.1° grids using the method introduced by *Duffie and Beckman* [1980] and then summing those estimates for each river basin. The Hargreaves method was the highest-ranked temperature-based method for computing PE reported by *Jensen et al.* [1990]. The Hargreaves method is the only temperature-based method recommended by *Shuttleworth* [1993].

3. Introduction to Climate Elasticity of Streamflow

Schaake [1990] introduced the concept of elasticity for evaluating the sensitivity of streamflow to changes in climate. Climate elasticity of streamflow can be defined by the proportional change in streamflow Q divided by the proportional

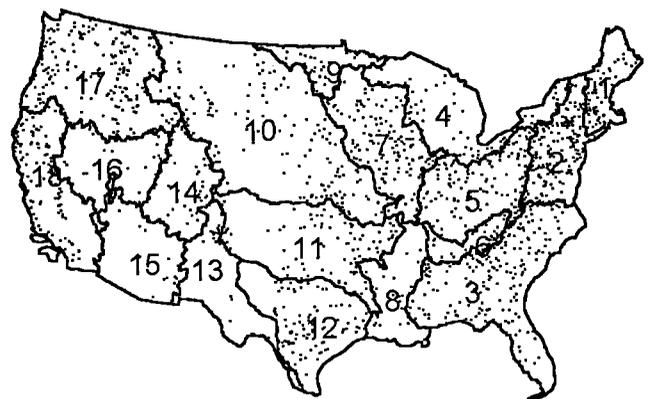
**Figure 1.** Location of streamgauges within each of the 18 major U.S. water resource regions [from *Slack et al.*, 1993].

Table 2. Climate and Streamflow Statistics for Sacramento, Animas, and Saline River Basins

Statistic	Symbol	River Basin		
		Sacramento	Animas	Saline
Mean annual Q , mm	μ_Q	991	397	414
Coefficient of variation of Q	C_Q	0.46	0.34	0.52
Mean annual P , mm	μ_P	1380	713	1312
Coefficient of variation of P	C_P	0.31	0.17	0.20
Mean annual PE, mm	μ_{PE}	1160	972	1381
Coefficient of variation of PE	C_{PE}	0.032	0.033	0.036
Correlation of Q and P	$\rho_{Q,P}$	0.98	0.88	0.93
Correlation of Q and PE	$\rho_{Q,PE}$	-0.50	-0.62	-0.52
Correlation of P and PE	$\rho_{P,PE}$	-0.51	-0.64	-0.64

change in a climatic variable such as precipitation P . Thus precipitation elasticity of streamflow is defined as

$$\varepsilon_P(P, Q) = \frac{dQ/Q}{dP/P} = \frac{dQ}{dP} \frac{P}{Q}. \quad (1)$$

Similar definitions exist for other climate variables such as potential evapotranspiration. Subsequent to its introduction to the water resources literature by *Schaake* [1990], *Dooge* [1992] and *Dooge et al.* [1999] termed $\varepsilon_P(P, Q)$ a sensitivity factor, and *Kuhnel et al.* [1991] termed it a magnification factor.

The economics literature contains discussions on the interpretation and estimation of elasticity which date back to the early twentieth century. *Lerner* [1933] discussed difficulties associated with arc elasticity defined as elasticity in (1) over a discrete range of the variables of interest. One difficulty with the estimation of elasticity is that it is often estimated from a model, and, of course, the form of the hydrologic model is always unknown.

The elasticity of power law models such as $Q = \alpha P^\beta$ is unique because for such models one can easily show that the precipitation elasticity $\varepsilon_P(P, Q) = \beta$ is constant, avoiding the need to define the range of P over which elasticity is meaningful. One is tempted to conclude from this result that all linear models ($\beta = 1$) exhibit an elasticity of unity $\varepsilon_P(P, Q) = 1$. To disprove this, consider the model $Q = P - \tau$, where τ is a constant with $\tau < P$. The precipitation elasticity for this model would be $\varepsilon_P(P, Q) = P/(P - \tau)$ so that the nonhomogeneous term τ causes the elasticity to depend on precipitation. Similarly, there is a problem in comparing temperature elasticity computed in different studies in which temperature is expressed in different units such as kelvins or degrees Fahrenheit. This is because transformations among units of temperature measurements involve nonhomogeneous terms. Comparisons of precipitation P and potential evapotranspiration PE elasticities are not hindered in this way, since conversions between SI units and U.S. customary units involve multiplicative transformations.

The precipitation elasticity of streamflow $\varepsilon_P(P, Q)$ defined in (1) is a random variable which depends on P and Q . In this study we focus on another definition of elasticity defined at the mean value of the climatic variable

$$\varepsilon_P(\mu_P, \mu_Q) = \left. \frac{dQ}{dP} \right|_{P=\mu_P} \frac{\mu_P}{\mu_Q}, \quad (2)$$

where μ_P and μ_Q denote mean values of precipitation and streamflow, respectively. The definition $\varepsilon_P(\mu_P, \mu_Q)$ in (2) is

the metric employed here for comparing the climate response of river basins.

4. Robust Estimators of Climate Elasticity: An Experiment

One goal of this study is to develop a contour map of $\varepsilon_P(\mu_P, \mu_Q)$ for the United States. A defensible map can only be made if the method of estimation of $\varepsilon_P(\mu_P, \mu_Q)$ is reliable and proven. In this section we describe a Monte Carlo experiment which evaluates the performance of alternative estimators of $\varepsilon_P(\mu_P, \mu_Q)$. To evaluate robustness of alternative estimators, it is necessary to generate climate and streamflow data from a variety of different model structures. In the following experiments we assume that the first- and second-order sample moments of observed streamflow and climate for three basins represent the true climate/streamflow regime. We chose the Sacramento River in California and the Animas River near Durango, Colorado, because previous hydroclimatologic studies exist for each of these basins. A third basin, the Saline River in Arkansas, was chosen because it has a particularly high value of $\varepsilon_P(\mu_P, \mu_Q)$. These three basins are the basis of truth for the following Monte Carlo experiments.

4.1. Description of Theoretical Hydroclimatologic Regimes

Previous hydroclimatologic investigations have been performed for the Sacramento River basin (e.g., *Jeton et al.* [1996], *Risbey and Entekhabi* [1996], and others) and the Animas River basin (e.g., *Nash and Gleick* [1991], *Revelle and Waggoner* [1983], *Schaake* [1990], and others). All hydroclimatologic records for these three basins were obtained for the period 1951–1988 from the databases described in section 2. Table 2 summarizes the assumed values of the moments of annual streamflow and climate for the Sacramento, Animas, and Saline basins. We found that PE can be well approximated by the bivariate linear model

$$PE = \mu_{PE} + \rho_{P,PE}(\sigma_{PE}/\sigma_P)(P - \mu_P) + \gamma, \quad (3)$$

where γ is normally distributed with zero mean and variance $\sigma_\gamma^2 = \sigma_{PE}^2 - \rho_{P,PE}^2 \sigma_P^2$. Note that the correlation of P and PE is -0.51 , -0.64 , and -0.64 for the Sacramento, Animas, and Saline basins, respectively.

4.2. Theoretical Hydroclimatologic Models of Annual Streamflow

In sections 4.2.1 through 4.2.2.2 we introduce a series of theoretical model structures for representing the relationship

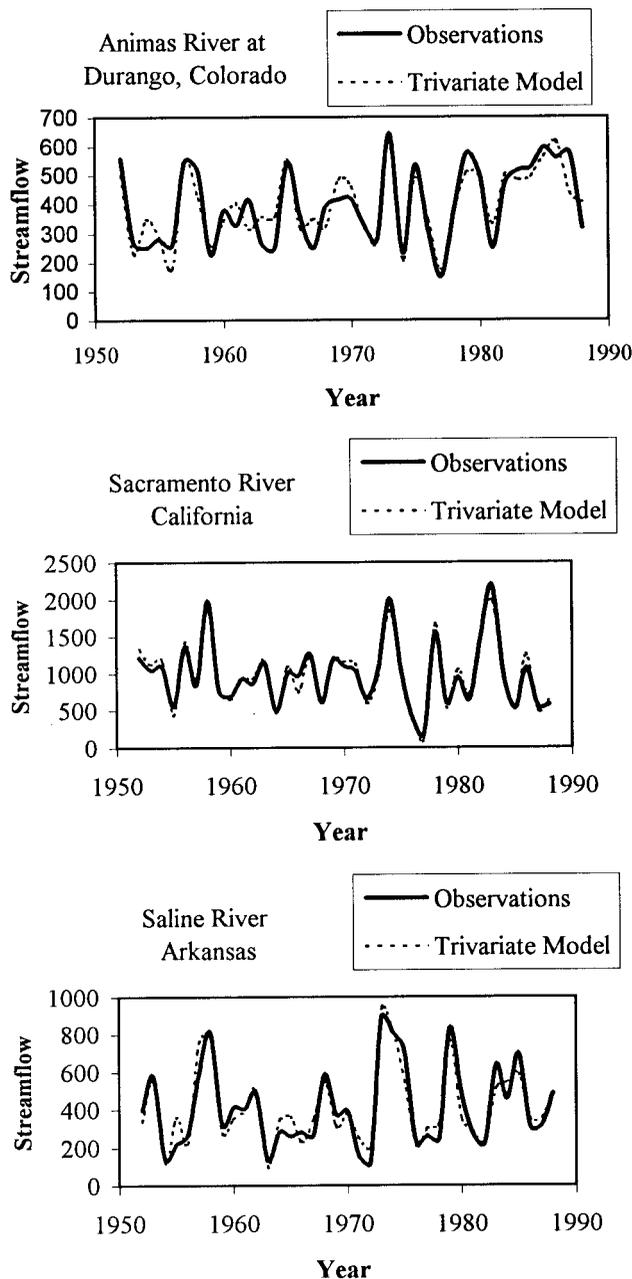


Figure 2. Comparisons of streamflow observations for Animas, Sacramento, and Saline River basins with streamflow estimated using the trivariate model.

between annual streamflow Q , annual precipitation P , and annual potential evapotranspiration PE. We introduce a linear statistical model and two nonlinear hydrologic models.

4.2.1. Statistical model. A simple linear statistical model is considered because it is able to approximate observed climate and streamflow data and because theoretical expressions are easily obtained for $\varepsilon_P(\mu_P, \mu_Q)$. A trivariate linear model relating Q to P and PE is

$$Q = \alpha + \beta P + \delta PE + \eta, \tag{4}$$

where α , β , and δ are model parameters and η are independent and identically distributed model errors with mean zero and constant variance σ_η^2 . For the trivariate model,

$$\varepsilon_P(\mu_P, \mu_Q) = \rho_{Q,P}(C_Q/C_P), \tag{5}$$

where $\rho_{Q,P}$ is the cross correlation of Q and P and C_Q and C_P are the coefficient of variation of Q and P , respectively. Note that (5) also holds for a bivariate model defined by (4) with $\delta = 0$. Tsai [1998] shows that $C_Q > C_P$, and is often significantly so, for all regions of the United States and that $\rho_{Q,P}$ is usually near unity. Hence we expect values of $\varepsilon_P(\mu_P, \mu_Q) > 1$ for this linear model. We use a nearly unbiased estimator introduced by Vogel et al. [1998] to estimate coefficient of variations. Figure 2 documents that the trivariate model can reproduce the historical behavior of annual streamflow for the Animas, Sacramento, and Saline River basins.

4.2.2. Hydrologic models. We consider two different nonlinear hydrologic models. It is tempting to employ a linear hydrologic model such as the “*abc*” model introduced by Fiering [1967] because it is very easy to derive its theoretical elasticity properties. For the *abc* model, $\varepsilon_P(\mu_P, \mu_Q) = 1$, yet it is well known that observed values of $\varepsilon_P(\mu_P, \mu_Q)$ are usually in excess of unity [Schaake, 1990; Vogel et al., 1999], and are often significantly so; hence we dispense with the *abc* model. We drop all linear water balance models from consideration because Schaake [1990] showed that the P and PE elasticity of linear watershed models cannot mimic the sensitivity of observed streamflow to climate change.

4.2.2.1. Nonlinear *abc* model: The *abc* model is a linear rainfall-runoff model introduced by Fiering [1967] which relates precipitation to evapotranspiration, groundwater storage, groundwater outflow, and streamflow using only precipitation as a model input and employing three model parameters a , b , and c . The *abc* model assumes that actual evapotranspiration $E_t = bP_t$ is a linear function of available moisture, where b is a model parameter and P_t is precipitation in time interval t . In reality, E_t depends on both available moisture and potential evapotranspiration. A number of empirical relationships have been introduced which express observed relationships between long-term actual evaporation E_t , potential evaporation PE $_t$, and precipitation P_t . Dooge [1992] and Kuhnel et al. [1991] summarize various relationships of this type. We employ one of those relationships termed the Turc-Pike equation:

$$E_t = \frac{P_t}{\sqrt{1 + (P_t/PE_t)^2}}. \tag{6}$$

Studies by Budyko [1974] for >1000 catchments in the USSR provide empirical support for the Turc-Pike relationship. The nonlinear annual *abc* model is created by replacing the term $E_t = bP_t$ in the *abc* model with the expression $E_t = bP_t/\sqrt{1 + (P_t/PE_t)^2}$. Precipitation elasticity for this model is easily derived analytically.

4.2.2.2. The “*abcd*” model: The *abcd* model is a nonlinear hydrologic model which accepts both P and PE as input, producing Q as output. Internally, the model also represents soil moisture storage, groundwater storage, groundwater outflow, and actual evapotranspiration. The *abcd* model was originally introduced by Thomas [1981] using an annual time step. The *abcd* model was later compared with numerous other water balance models leading to its recommendation by Alley [1984]. Vandewiele et al. [1992] also found that the *abcd* model compares favorably with several other water balance models. The primary difference between the nonlinear *abc* model described above and the *abcd* model is that the *abcd* model includes a soil moisture storage term; hence actual evapotranspiration depends on the available soil moisture as well as on PE. A series

Table 3. Summary of Properties of Statistical and Hydrologic Models Fit to Three River Basins

Statistic	Models								
	Sacramento River, California			Animas River, Colorado			Saline River, Arkansas		
	Trivariate	<i>abc</i>	<i>abcd</i>	Trivariate	<i>abc</i>	<i>abcd</i>	Trivariate	<i>abc</i>	<i>abcd</i>
α or <i>a</i>	-454	0.0084	0.999	106	0.001	0.971	-1496	0.017	0.999
β or <i>b</i>	1.06	0.496	413	0.933	0.555	348	0.852	0.962	1550
<i>c</i>	PNIM ^a	0.988	0.0181	PNIM	0.984	0.0741	PNIM	0.650	0.353
δ or <i>d</i>	-0.012	PNIM	0.68	-0.384	PNIM	0.695	0.5741	PNIM	0.999
σ_η/μ_Q	0.095	0.115	0.097	0.159	0.174	0.159	0.184	0.199	0.149
R^2 , %	95.5	96.3	95.6	78.1	78.6	78.6	87.5	86.5	91.8
$\varepsilon_P(\mu_P, \mu_Q)$	1.47	1.23	1.40	1.79	1.30	1.78	2.48	2.18	3.30

^aPNIM, parameter not in model.

of chain rule computations were used to derive $\varepsilon_P(\mu_P, \mu_Q)$ (see Sankarasubramanian [2001] for details).

4.3. Summary of Goodness-of-Fit of Linear and Nonlinear Models

Each model was fit to the observations of Q , P , and PE for the three basins. The trivariate linear model was fit using the method of moments. The nonlinear *abc* model and the *abcd* model were fit using ordinary least squares, with constraints added to preserve all moments reported in Table 2. Table 3 summarizes the model parameters, standard error of the model errors, R^2 , and $\varepsilon_P(\mu_P, \mu_Q)$. The standard errors reported in Table 3, σ_η/μ_Q , are standardized by the mean of the dependent variable μ_Q to enable comparisons between basins. Table 3 reveals that the standard error of the residuals is always less than 12%, 18%, and 20% of the mean for the Sacramento, Animas, and Saline basins, respectively. The three models led to roughly equivalent fits for all three basins.

Schaake [1990] estimated $\varepsilon_P(\mu_P, \mu_Q)$ for the Animas basin as 1.97 using the NWSRFS. Results from Table 3 show that the precipitation elasticity estimated by the trivariate model (1.79) and the *abcd* model (1.78) are fairly close to Schaake's [1990] estimate of 1.97. However, the nonlinear *abc* model yields quite different results (1.30). The results in Table 3 demonstrate how sensitive climate elasticity is to the assumed model structure. The precipitation elasticity given in Table 3 for the Sacramento basin is in agreement with the results documented by Risbey and Entekhabi [1996].

4.4. Monte Carlo Experiments

One goal of this study is to develop a robust estimator of climate elasticity to be used with annual climate and streamflow data. The developed estimator should perform well, regardless of the structure of the model describing the relationship between Q , P , and PE. In this section we summarize an experiment which evaluates the performance of alternative elasticity estimators when streamflows are generated by each of the three models posed in section 4.2. All experiments assume that the statistics reported in Tables 2 and 3 represent the true hydrologic structure for these three basins.

4.4.1. Experimental design. Using the linear trivariate, nonlinear *abc*, and *abcd* models, we generate 10,000 traces of P , PE, and Q , each of length $n = 50$ for the Animas, Sacramento, and Saline basins. Generation of 10,000 traces enables us to evaluate the bias and root-mean-square error associated with each estimator of climate elasticity. Generation of annual climate and streamflow traces from three different theoretical

models enables us to evaluate the robustness of various elasticity estimators introduced in section 4.4.3.

4.4.2. Generation of climate and streamflow traces. Sequences of P are generated from a gamma distribution since Guttman *et al.* [1993] found the gamma distribution to provide an acceptable fit to sequences of annual precipitation across the entire United States. Tsai [1998] and others have found that sequences of annual precipitation in the United States are approximately serially independent; hence that assumption is used here. Estimates of PE are obtained from (3) using the generated P values along with the assumed theoretical moments in Table 2. This insures that the P and PE sequences reproduce the observed correlation structure summarized in Table 2. Using the generated sequences of P and PE, synthetic sequences of Q are then generated from each of the three theoretical models. In order to reproduce all the moments listed in Table 2, it was necessary to add normally distributed model error with zero mean and variance σ_η^2 (from Table 2). If we did not add model error, the synthetic streamflows would not reproduce the theoretical properties of the assumed theoretical river basins listed in Table 2.

4.4.3. Estimators of climate elasticity. In this section we describe the estimators of $\varepsilon_P(\mu_P, \mu_Q)$ to be evaluated in the Monte Carlo experiments. In each case an analogous PE elasticity estimator exists. However, we do not report those results here. A natural nonparametric approach to estimation of $\varepsilon_P(\mu_P, \mu_Q)$ in (2) is to use

$$e_P^1 = \text{median} \left(\frac{Q_t - \bar{Q}}{P_t - \bar{P}} \frac{\bar{Q}}{\bar{P}} \right), \quad (7)$$

where \bar{Q} and \bar{P} are the long-term sample means. Limbrunner [1998] used this nonparametric estimator to summarize the regional behavior of P and PE elasticity across the United States. There is a numerical problem with this estimator when P_t approaches \bar{P} , causing e_P^1 to approach infinity.

The second estimator is obtained from the trivariate model in (4), based on the result in (5):

$$e_P^2 = \hat{\rho}_{Q,P}(\hat{C}_Q/\hat{C}_P). \quad (8)$$

We term (8) the bivariate estimator because the trivariate model reduces to the bivariate model for the case of P elasticity. Circumflexes over variables imply a sample estimate of the indicated parameter.

The power law model $Q = \alpha P^\beta$ exhibits a fixed value of $\varepsilon_P(P, Q) = \beta$. Since none of the assumed models exhibit a power law structure, it is instructive to consider a power law

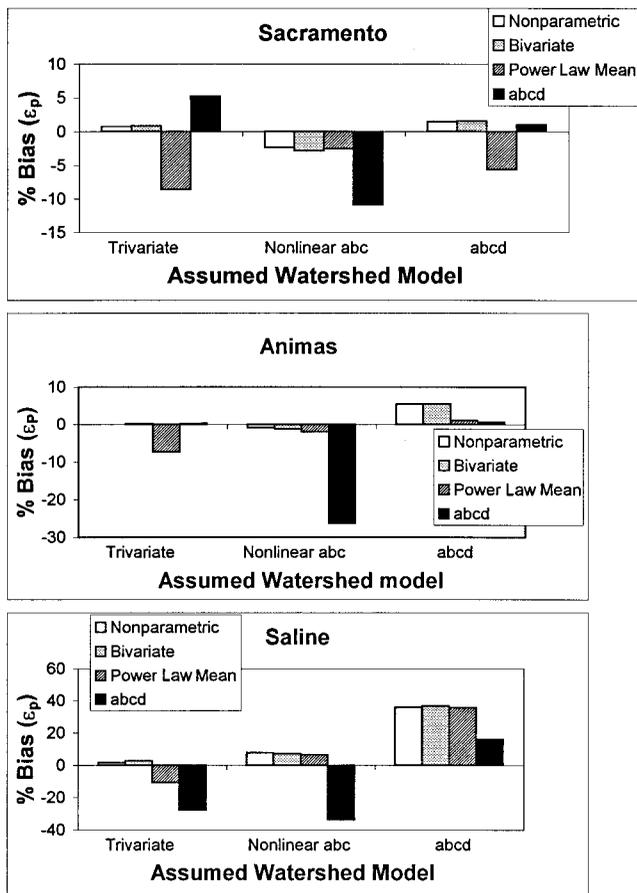


Figure 3. The percent bias of four estimators of precipitation elasticity of streamflow for the Sacramento, Animas, and Saline basins.

estimator; because if it performed well in this experiment, it would certainly be a robust estimator. We consider the estimator of $\epsilon_P(\mu_P, \mu_Q)$:

$$e_P^3 = \hat{\alpha}\hat{\beta}(\hat{\mu}_P/\hat{\mu}_Q), \tag{9}$$

which we term the power law mean estimator. Here $\hat{\beta}$ and $\hat{\alpha}$ are the least squares estimators of a power law model.

Finally, we consider estimates of climate elasticity based on the *abcd* water balance model. Estimation of climate elasticity using the *abcd* model requires calibration of the model using the generated traces of Q , P , and PE. The *abcd* model is calibrated using the shuffled complex evolution (SCE-UA) algorithm [Duan *et al.*, 1992]. The SCE-UA algorithm is a global optimization algorithm developed at the University of Arizona and is widely used for automatic calibration of watershed models.

In addition to the above estimation methods, Sankarasubramanian [2001] reports the performance of many other estimators including several localized nonparametric regression methods designed particularly for estimation of derivatives [Loader, 1999]. However, none of those methods, including the more advanced estimators, offered improvements over the estimators reported here.

4.4.4. Performance evaluation measures. A primary goal is the development of a contour map illustrating the sensitivity of streamflow to climate; hence an approximately unbiased

estimator of $\epsilon_P(\mu_P, \mu_Q)$ is needed. The optimal performance of the alternative estimators of $\epsilon_P(\mu_P, \mu_Q)$ is evaluated using the performance measure percent bias defined by

$$\text{percent bias}^k(e_P^j) = \frac{\epsilon_P^k - E(e_P^j)}{\epsilon_P^k} 100, \tag{10}$$

where $k = 1$ to 3 denotes each one of the three assumed watershed models, $i = 1$ to 4 denotes the four elasticity estimators introduced in section 4.4.3, ϵ_P^k denotes the true value of $\epsilon_P(\mu_P, \mu_Q)$ given in Table 3 for the k th model, and e_P^i denotes the vector of 10,000 estimates of $\epsilon_P(\mu_P, \mu_Q)$ using the i th estimator.

4.4.5. Results. Figure 3 summarizes our Monte Carlo experiments by illustrating the percent bias of four estimators of $\epsilon_P(\mu_P, \mu_Q)$ for the basins of the Sacramento, Animas, and Saline Rivers. None of the estimators are uniformly superior in terms of percent bias for all three model structures. The *abcd* model estimator performs poorly when flows arise from either a nonlinear *abc* or trivariate model. The three remaining estimators perform roughly equivalently. However, since we favor a nonparametric estimator over a parametric estimator, we recommend the nonparametric estimator e_P^1 for general usage. Comparisons of the root-mean-square error of these four estimators did not lead to any additional useful information; hence they are not reported here.

5. Climate Elasticity of Streamflow in the United States

5.1. Elasticity Map Using Nonparametric Estimation

In this section the robust nonparametric estimator of climate elasticity identified in section 4.4.5 is employed to estimate contour maps of $\epsilon_P(\mu_P, \mu_Q)$ for the United States. Schaake [1990] summarized the sensitivity of streamflow to changes in P and PE in the form of contour maps for the southeastern United States. His maps were obtained by perturbing a nonlinear monthly water balance model. Risbey and Entekhabi [1996] created contour maps of the percent change in streamflow versus the percent change in precipitation and temperature for the Sacramento basin.

Precipitation elasticity estimates for the 1291 HCDN river basins are obtained using the nonparametric estimator e_P^1 given in (7). Note that e_P^1 represents the median of annual estimates of climate elasticity over the period 1951–1988 at each site. We used Kendall’s tau test to document that estimates of e_P^1 are independent of P at each site. This suggests that P elasticities are fixed for each basin, so we summarized $\epsilon_P(\mu_P, \mu_Q)$ using the median values as defined in (7).

Figure 4 illustrates a contour map of $\epsilon_P(\mu_P, \mu_Q)$ for the continental United States. A value >1 indicates that a 1% change in precipitation can cause a $>1\%$ change in streamflow. Occasionally, such as in portions of Montana and North Dakota, values of P elasticity are <1 . Other than these two regions, the P elasticity for the entire United States ranges from 1.0 to 2.5. The contours in Figure 4 are very similar to the water balance model-based contour plots developed by Schaake [1990] for the southeastern United States with values in the range of 2.0–2.5. Section 5.4 of this paper provides a physical interpretation for the variations in $\epsilon_P(\mu_P, \mu_Q)$ across the United States.

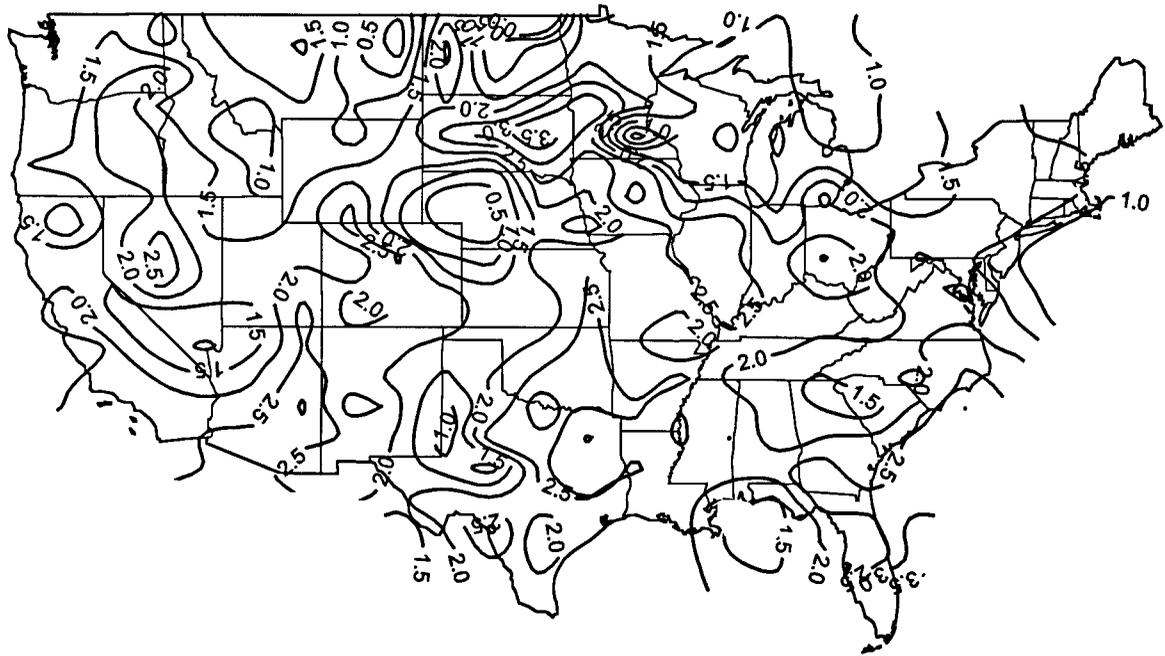


Figure 4. Contour map of precipitation elasticity of streamflow for the continental United States.

5.2. Model-Based Estimates of Climate Elasticity in the United States

It is instructive to compare the performance of nonparametric estimates of $\epsilon_P(\mu_P, \mu_Q)$ with estimates based on a water-

shed model. Such comparisons can provide insights into the impact of model calibration and model choice on estimated climate sensitivity of streamflow. The *abcd* model is calibrated to 30-year annual time series of *P*, *PE*, and *Q* at each of the

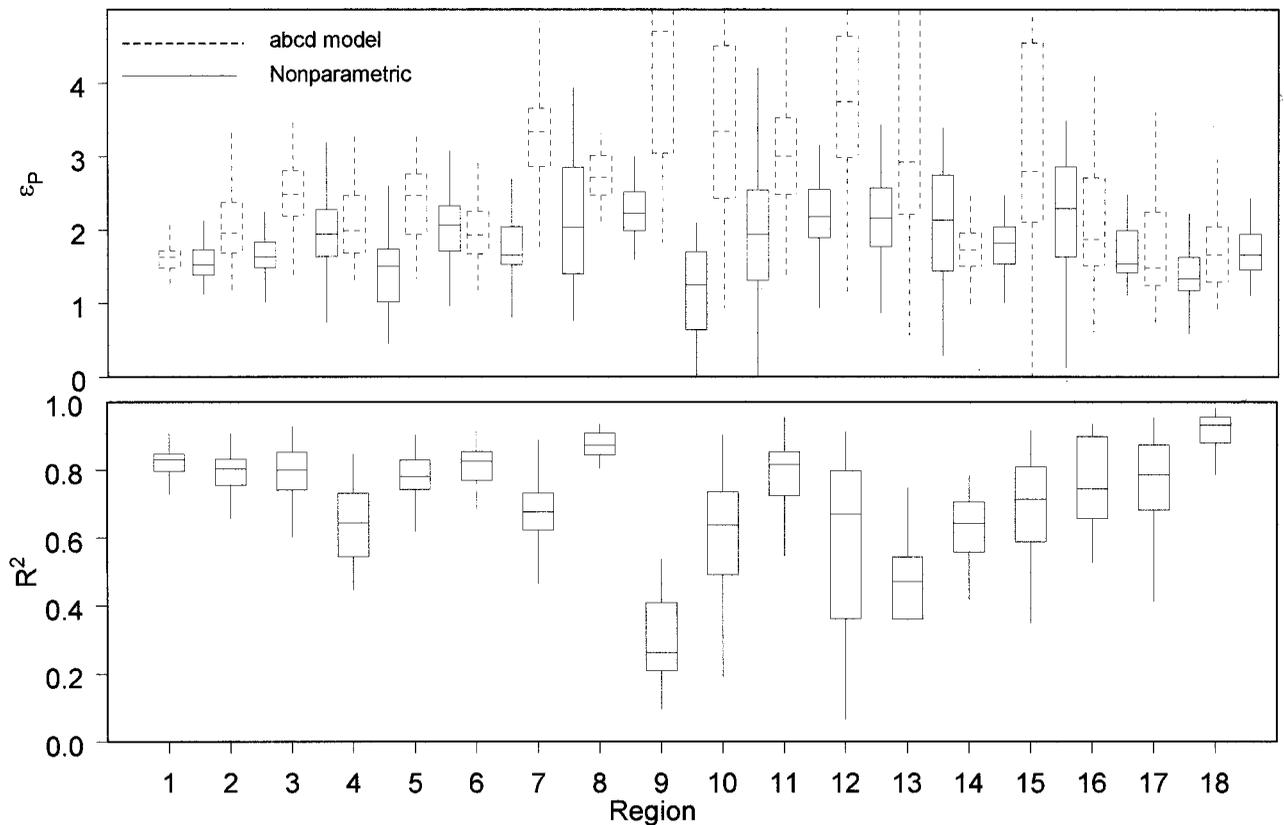


Figure 5. Comparison of boxplots of nonparametric and “abcd” model-based estimates of *P* elasticity of streamflow for the 18 major U.S. water resource regions.

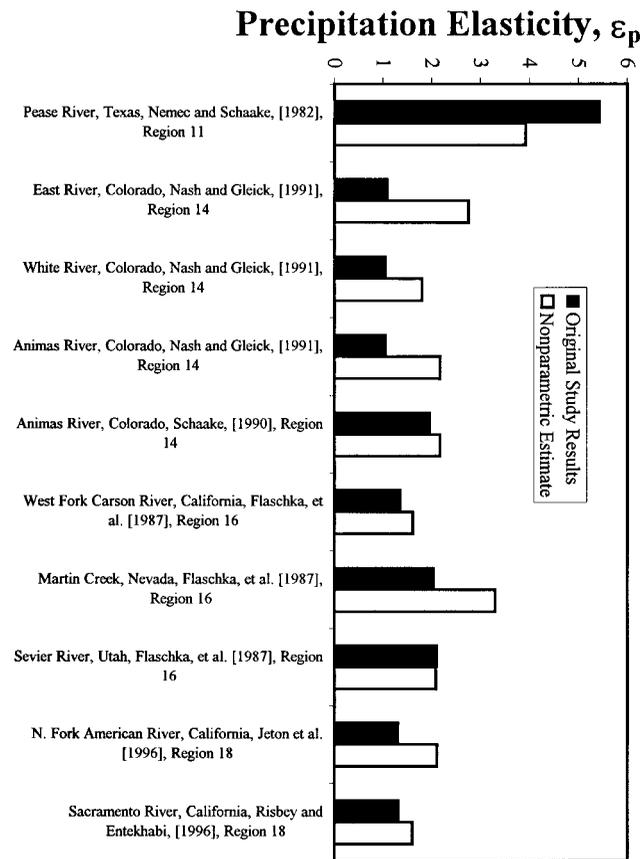


Figure 6. Comparisons of precipitation elasticity of streamflow from this study with 10 previous studies.

1291 basins shown in Figure 1, using the SCE-UA algorithm [Duan *et al.*, 1992]. Estimates of $\varepsilon_P(\mu_P, \mu_Q)$ were obtained by substituting the calibrated model parameters and hydrologic state variables into analytic expressions for $\varepsilon_P(\mu_P, \mu_Q)$ derived by Sankarasubramanian [2001]. The bottom graph of Figure 5 illustrates box plots of the goodness-of-fit statistic R^2 corresponding to the fit of the *abcd* model to the observed streamflow and climate data for the 18 water resource regions shown in Figure 1. Overall, the goodness of fit is quite good with the exception of the midwestern regions of the United States. The top graph of Figure 5 compares box plots of $\varepsilon_P(\mu_P, \mu_Q)$ obtained using a nonparametric estimator and the *abcd* model. In general, the agreement between these two methods is quite good, with the exception of the midwestern regions where the goodness of fit of the *abcd* model is poor. In these midwestern regions the *abcd* model often led to estimates of $\varepsilon_P(\mu_P, \mu_Q)$ in excess of 3. Figure 5 illustrates clearly that these very high values of climate elasticity are probably suspect because they correspond to situations when the goodness-of-fit statistic R^2 was very low. Thus Figure 5 demonstrates that poor model calibrations can lead to suspect estimates of $\varepsilon_P(\mu_P, \mu_Q)$.

5.3. Comparisons With Other Studies

In this section we compare the results of our nonparametric estimator with results from 10 detailed modeling studies. Figure 6 summarizes comparisons of the nonparametric elasticity estimator e_P^1 with estimates derived from 10 river basin studies performed by Nemece and Schaake [1982], Nash and Gleick

[1991], Schaake [1990], Flaschka *et al.* [1987], Jeton *et al.* [1996], and Risbey and Entekhabi [1996]. Each of these studies reported simulated values of annual streamflow Q corresponding to various values of annual precipitation. Most studies reported increases in annual streamflow which result from arbitrary increases (e.g., +10% or +25%) and decreases in precipitation. To compute $\varepsilon_P(\mu_P, \mu_Q)$, we fit a power law model to the reported values of Q and P . In all cases a power law model gave a nearly perfect fit, with R^2 values always $>99.5\%$. We only considered modeled results where PE was held constant at its historical average value. The agreement is generally quite good with the exception of the three studies by Nash and Gleick [1991].

A comparison of precipitation elasticity estimates for the Animas basin obtained from this study with estimates given by others (see Table 1 and Figure 6) reveals that the nonparametric estimator $e_P^1 = 1.83$ is in good agreement with the estimates 1.97 and 1.90 by Schaake [1990] and Vogel *et al.* [1999], respectively. However, $e_P^1 = 1.83$ is in poor agreement with the estimates 1.09 and 1.05 from Nash and Gleick [1991] and Revelle and Waggoner [1983], respectively. Since we now know (from the Monte Carlo study) that the nonparametric estimator in (7) is robust and roughly unbiased, this leads us to suspect the results of the studies by Nash and Gleick [1991] and Revelle and Waggoner [1983]. Apparently, this nonparametric elasticity estimator may be quite useful for validating hydroclimatologic investigations. One is tempted to conclude that the more detailed monthly and daily simulation studies cited here lead to climate elasticities which are closer to the truth than this study which uses only annual data. However, even those studies can disagree significantly, as is the case for the Animas River. Schaake's [1990] results are grossly different from those of Nash and Gleick [1991] for the Animas basin, yet Schaake's [1990] results are very close to our own. Apparently, model calibration plays a significant role in determining the sensitivity of streamflow to climate because calibration is the primary difference between the studies by Nash and Gleick [1991] and Schaake [1990].

5.4. Physical Interpretation of Variations in Climate Elasticity of Streamflow

Dooge *et al.* [1999] use a soil moisture accounting model, similar to the *abcd* model used in this study, along with three different climate forcing functions to derive analytical relationships between $\varepsilon_P(\mu_P, \mu_Q)$ and physical characteristics of catchments. Ideally, analytic physical relationships for ε_P of the type described by Dooge *et al.* [1999] would be used to provide a physical basis for the variations in $\varepsilon_P(\mu_P, \mu_Q)$ illustrated in Figure 4. Instead, in this initial study we report some of the more intuitively obvious physical mechanisms which appear to control the climate sensitivity of streamflow.

5.4.1. Impact of snowpack storage. Figure 7 illustrates that $\varepsilon_P(\mu_P, \mu_Q)$ is quite low (1.0–1.5) in regions where snow storage is significant such as in the Rocky Mountains, Great Lakes, and New England region. These regions have significant snow storage which acts to buffer the impacts of climate change. This effect is illustrated in Figure 7 which plots regional average values of P elasticity versus regional average snowpack depth for the 18 water resource regions shown in Figure 1. Figure 7 demonstrates that $\varepsilon_P(\mu_P, \mu_Q)$ is lower in regions with higher average annual snowpack depths. Risbey and Entekhabi [1996] hypothesize that snow storage buffers

annual streamflow quantities by changing streamflow timing in response to changes in climate.

5.4.2. An equilibrium interpretation of the climate elasticity of streamflow. A common equilibrium assumption in watershed hydrology is that the continuity equation can be expressed as

$$\bar{Q} = \bar{P} - \bar{E}, \tag{11}$$

where overbars denote long-term mean values. Assumptions implicit in (11) include that groundwater seepage into and out of the basin cancel each other and long-term changes in basin storage are negligible. *Kuhnel et al.* [1991], *Dooge* [1992], and others combine (11) with equations of the form

$$\bar{E}/\bar{PE} = \Phi[\bar{P}/\bar{PE}] = \Phi[\phi], \tag{12}$$

where Φ represents a functional transformation between the variables \bar{E}/\bar{PE} and \bar{P}/\bar{PE} , to obtain general relationships for the equilibrium sensitivity of streamflow to climate. Here the ratio $\phi = \bar{P}/\bar{PE}$ is termed the humidity ratio. *Dooge* [1992] refers to (12) as the Budyko hypothesis. *Kuhnel et al.* [1991] show that (11) and (12) imply that

$$\varepsilon_P(P, Q) + \varepsilon_{PE}(PE, Q) = 1, \tag{13}$$

so that the sum of P elasticity of streamflow and PE elasticity of streamflow is unity. *Dooge* [1992] found that ε_P is particularly sensitive to the way in which subsurface drainage is modeled, so that one cannot expect (11)–(13) to be representative of most catchments. Nevertheless, it provides an elegant and simple conceptual framework from which to begin our interpretations of climate elasticity.

Kuhnel et al. [1991] and *Dooge* [1992] summarize three commonly used functions which represent the Budyko hypothesis. They include the Ol'dekop function $\bar{E}/\bar{PE} = \tanh(\phi)$, the Schreiber function $\bar{E}/\bar{PE} = \phi[1 - \exp(-1/\phi)]$, and the Turc-Pike function given in (6).

Figure 8 illustrates nonparametric estimates of $\varepsilon_P(\mu_P, \mu_Q)$ for all HCDN basins in regions 1, 3, 10, 12, and 17 versus their humidity index $\phi = \bar{P}/\bar{PE}$. These five regions reflect a very broad range of climate conditions in the United States. Also shown for comparison are derived theoretical relationships corresponding to the use of (11) along with the Turc-Pike, Schreiber, and Ol'dekop equations. None of the theoretical

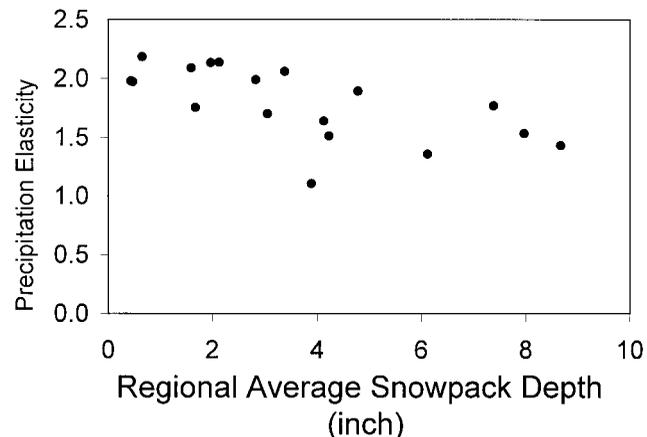


Figure 7. Regional average precipitation elasticity versus regional average snow depth.

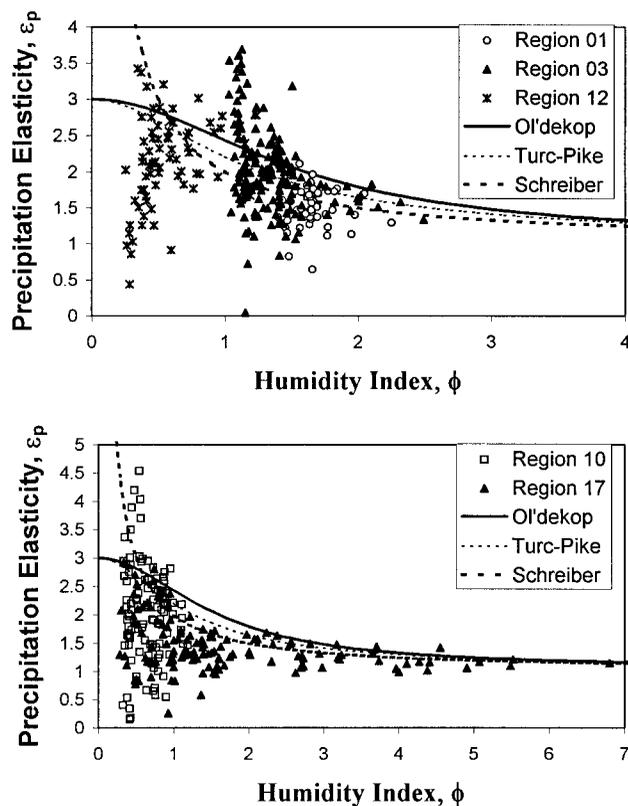


Figure 8. Precipitation elasticity of streamflow $\varepsilon_P(\mu_P, \mu_Q)$ as a function of the humidity index $\phi = \bar{P}/\bar{PE}$ for basins in regions 1, 3, 10, 12, and 17.

relations adequately represent the elasticity behavior of catchments. This is to be expected from *Dooge et al.* [1999], who show that $\varepsilon_P(\mu_P, \mu_Q)$ depends on many factors in addition to the humidity ratio such as the stochastic nature of climate, field capacity of soils, soil moisture levels, length of soil water depletion, and saturated hydraulic conductivity. Figure 8 reveals that only for very humid regions ($\phi > 2$), such as those in the Northwest, does the Budyko hypothesis provide generally good agreement with empirical observations. Figure 8 also shows that humid basins tend to have significantly lower values of $\varepsilon_P(\mu_P, \mu_Q)$ and tend to be much more homogeneous in terms of $\varepsilon_P(\mu_P, \mu_Q)$ than arid regions.

Figure 9 illustrates estimates of dQ/dP for 1291 basins using the nonparametric estimator e_P^1 in (7) versus the humidity index $\phi = \bar{P}/\bar{PE}$ and the ratio σ_Q/σ_P which represents the variability of streamflow in comparison to the variability of precipitation. Also shown for comparison in Figure 9 are theoretical relations corresponding to the Budyko hypothesis. Figure 9 uses dQ/dP as its ordinate instead of $\varepsilon_P(\mu_P, \mu_Q)$ in order to avoid the problem of spurious correlation. Also shown in Figure 9 is an index of seasonality which provides a measure of the degree to which moisture (precipitation) and energy (temperature) are in phase or out of phase with each other. To quantify this issue, we compute the correlation between monthly temperature and precipitation ρ_{PQ} , as did *Wolock and McCabe* [1999]. In Figure 9 we assume that when $\rho_{PQ} > 0$, moisture and energy are “in phase,” and when $\rho_{PQ} < 0$, moisture and energy are “out of phase.” Interestingly, the variable σ_Q/σ_P alone explains 83% of the variability of dQ/dP . We also observe from Figure 9 that the Budyko hypothesis

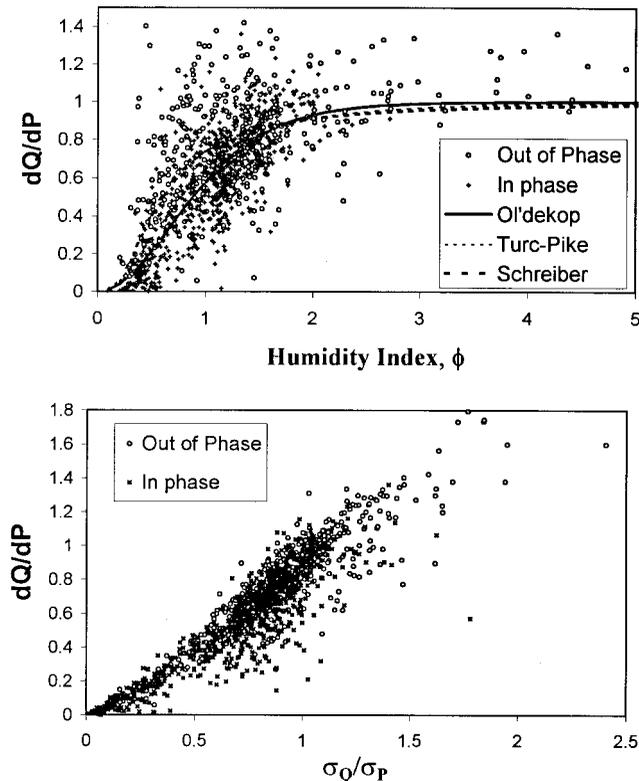


Figure 9. Derivative dQ/dP as a function of the humidity index $\phi = \bar{P}/\bar{PE}$ and σ_Q/σ_P .

holds best for “inphase” climates. Basins with “out of phase” climates tend to produce the highest values of dQ/dP and thus the highest values of $\varepsilon_P(\mu_P, \mu_Q)$.

6. Conclusions

This study has sought to develop a generalized understanding of the sensitivity of streamflow to changes in precipitation in the United States. The concept of elasticity ε introduced to the hydrological literature by *Schaake* [1990] is used to quantify the sensitivity of streamflow to changes in climate. This study and others [e.g., *Dooge*, 1992] have demonstrated that ε is highly model-dependent. This makes it difficult to generalize our understanding of ε because our understanding is also model-dependent. A Monte Carlo experiment compared various methods for estimation of the precipitation elasticity of streamflow ε_P . The preferred nonparametric estimator was used to create a map of ε_P for the United States, and some physical interpretations were provided for observed spatial variations in ε_P . The following conclusions were reached:

1. Both model form and model calibration play an important role in determining the sensitivity of model streamflow to climate. Estimates of ε_P were shown to depend on both model choice and model calibration. Therefore it is difficult, if not impossible, to estimate the sensitivity of streamflow to climate using a single watershed model.

2. Experiments reveal that the nonparametric estimator e_P^1 in (7) is useful, has low bias, and is as robust as or more robust than watershed model-based approaches for evaluating the sensitivity of streamflow to climate. The nonparametric approach does not require a model assumption or a calibration strategy.

3. The contour map in Figure 4 illustrates the spatial variability of $\varepsilon_P(\mu_P, \mu_Q)$. Values of $\varepsilon_P(\mu_P, \mu_Q)$ generally range from 1.0 to 2.5. The highest values of $\varepsilon_P(\mu_P, \mu_Q) \geq 2$ occur primarily in the arid and semiarid regions of the Midwest and Southwest.

4. Variations in $\varepsilon_P(\mu_P, \mu_Q)$ were shown to depend on snow storage, the timing of the balance between moisture and energy, and the humidity index $\phi = \bar{P}/\bar{PE}$. Basins with a very high humidity index such as in the Northwest were shown to have relatively constant and low values of $\varepsilon_P(\mu_P, \mu_Q)$. Similarly, basins with large snow storage had the lowest values of $\varepsilon_P(\mu_P, \mu_Q)$. The greatest spatial variation in $\varepsilon_P(\mu_P, \mu_Q)$ seemed to occur in regions where moisture and energy are out of phase with each other.

5. Comparisons of values of $\varepsilon_P(\mu_P, \mu_Q)$ across studies indicated that variations can be quite significant and are due to differences in model form, model calibration, and input data. The nonparametric estimator of $\varepsilon_P(\mu_P, \mu_Q)$ introduced here is shown to be a useful validation statistic. Hopefully, future climate change investigations will compare their results with our results, illustrated in Figure 4.

6. The Budyko hypothesis given in (12) provides only a very rough approximation of the sensitivity of streamflow to climate change. The Budyko hypothesis is most useful for basins with humidity ratios >1 and for basins in which the moisture and energy balance are in phase with one another.

Acknowledgments. Although the research described in this article has been funded in part by the United States Environmental Protection Agency through STAR grant R 824992-01-0 to Tufts University, it has not been subjected to the Agency’s required peer and policy review and therefore does not necessarily reflect the views of the Agency, and no endorsement should be inferred. The authors are indebted to Dara Entekhabi and John Schaake for their comments and useful suggestions during the early phases of this research. The authors are indebted to Chris Daly for providing us with the monthly time series grids of temperature and precipitation and to Ian Wilson for his assistance in processing those grids. The authors also appreciate the helpful review comments of the anonymous reviewers.

References

- Alley, W. M., On the treatment of evapotranspiration, soil moisture accounting, and aquifer recharge in monthly water balance models, *Water Resour. Res.*, 20(8), 1137–1149, 1984.
- Budyko, M. I., *Climate and Life*, edited by D. H. Miller, Academic, San Diego, Calif., 1974.
- Daly, C., R. P. Neilson, and D. L. Phillips, A statistical-topographic model for mapping climatological precipitation over mountainous terrain, *J. Appl. Meteorol.*, 33(2), 140–158, 1994.
- Dooge, J. C. I., Sensitivity of runoff to climate change: A Hortonian approach, *Bull. Am. Meteorol. Soc.*, 73(12), 2013–2024, 1992.
- Dooge, J. C. I., M. Bruen, and B. Parmentier, A simple model for estimating the sensitivity of runoff to long-term changes in precipitation without a change in vegetation, *Adv. Water Resour.*, 23, 153–163, 1999.
- Duan, Q., S. Sorooshian, and V. Gupta, Effective and efficient global optimization for conceptual rainfall-runoff models, *Water Resour. Res.*, 28(4), 1015–1031, 1992.
- Duffie, J. A., and W. A. Beckman, *Solar Engineering of Thermal Processes*, pp. 1–109, John Wiley, New York, 1980.
- Fiering, M. B., *Streamflow Synthesis*, Harvard Univ. Press, Cambridge, Mass., 1967.
- Flaschka, I., C. W. Stockton, and W. R. Boggess, Climatic variation and surface water resources in the great basin region, *Water Resour. Bull.*, 23(1), 47–57, 1987.
- Guttman, N. B., J. R. M. Hosking, and J. R. Wallis, Regional precipitation quantile values for the continental United States computed from L -moments, *J. Clim.*, 6(12), 2326–2340, 1993.

- Hargreaves, G. H., and Z. A. Samani, Estimating potential evapotranspiration, *J. Irrig. Drain. Eng.*, 108(3), 225–230, 1982.
- Jensen, M. E., R. D. Burman, and R. G. Allen, Evapotranspiration and irrigation water requirements, *ASCE Manuals and Rep. on Eng. Pr.* 70, 360 pp., Am. Soc. of Civ. Eng., Reston, Va., 1990.
- Jeton, A. E., M. D. Dettinger, and J. L. Smith, Potential effects of climate change on streamflow, eastern and western slopes of the Sierra Nevada, California and Nevada, *U.S. Geol. Surv. Water Resour. Invest. Rep.*, 95-4260, 60 pp., 1996.
- Kuhnel, V., J. C. I. Dooge, J. P. J. O'Kane, and R. J. Romanowicz, Partial analysis applied to scale problems in surface moisture fluxes, *Surv. Geophys.*, 12, 221–247, 1991.
- Leavesley, G. H., Modeling the effects of climate change on water resources, *Clim. Change*, 28(1/2), 159–177, 1994.
- Lerner, A. P., The diagrammatical representation of elasticity of demand, *Rev. Econ. Stud.*, 1, 39–44, 1933.
- Limbrunner, J. F., Climatic elasticity of streamflow in the United States, M.S. thesis, 39 pp., Tufts Univ., Medford, Mass., 1998.
- Loader, C., *Local Regression and Likelihood*, Springer-Verlag, New York, 1999.
- Nash, L. L., and P. H. Gleick, Sensitivity of streamflow in the Colorado basin to climatic changes, *J. Hydrol.*, 125, 221–241, 1991.
- Nemec, J., and J. Schaake, Sensitivity of water resource systems to climate variation, *Hydrol. Sci. J.*, 27, 327–343, 1982.
- Revelle, R. R., and P. E. Waggoner, Effects of a carbon dioxide-induced climatic change on water supplies in the western United States, in *Changing Climate*, pp. 419–432, Nat. Acad., Washington, D. C., 1983.
- Risbey, J. S., and D. Entekhabi, Observed Sacramento basin streamflow response to precipitation and temperature changes and its relevance to climate impact studies, *J. Hydrol.*, 184, 209–223, 1996.
- Sankarasubramanian, A., Hydroclimatology of the United States, Ph.D. dissertation, Tufts Univ., Medford, Mass., 2001.
- Schaake, J. C., From climate to flow, in *Climate Change and U.S. Water Resources*, edited by P. E. Waggoner, chap. 8, pp. 177–206, John Wiley, New York, 1990.
- Shuttleworth, W. J., Evaporation, in *Handbook of Hydrology*, edited by D. R. Maidment, p. 4.18, McGraw-Hill, New York, 1993.
- Slack, J. R., A. M. Lumb, and J. M. Landwehr, Hydroclimatic data network (HCDN): A U.S. Geological Survey streamflow data set for the United States for the study of climate variation, 1874–1988, *U.S. Geol. Surv. Water Resour. Invest. Rep.* [CD-ROM], 93-4076, 1993.
- Thomas, H. A., Improved methods for national water assessment, report contract WR 15249270, U.S. Water Resour. Council, Washington, D. C., 1981.
- Tsai, Y. E., The long-term persistence of annual streamflow and precipitation in the United States, M.S. thesis, 72 pp., Tufts Univ., Medford, Mass., Feb. 1998.
- Vandewiele, G. L., C.-Y. Xu, and Ni-Lar-Win, Methodology and comparative study of monthly water balance models in Belgium, China and Burma, *J. Hydrol.*, 134, 315–347, 1992.
- Vogel, R. M., Y. Tsai, and J. F. Limbrunner, The regional persistence and variability of annual streamflow in the United States, *Water Resour. Res.*, 34(12), 3445–3459, 1998.
- Vogel, R. M., I. Wilson, and C. Daly, Regional regression models of annual streamflow for the United States, *J. Irrig. Drain. Eng.*, 125(3), 148–157, 1999.
- Wolock, D. M., and G. M. McCabe, Explaining spatial variability in mean annual runoff in the conterminous United States, *Clim. Res.*, 11, 149–159, 1999.

J. Limbrunner, Haley and Aldrich, Inc., 340 Granite St., 3rd Floor, Manchester, NH 03102. (JFL@haleyaldrich.com)

A. Sankarasubramanian and R. Vogel, Department of Civil and Environmental Engineering, Tufts University, Medford, MA 02155. (sarumuga@tufts.edu; rvogel@tufts.edu)

(Received January 14, 2000; revised August 9, 2000; accepted October 17, 2000.)

