Applied Geography xxx (2010) 1–8

Contents lists available at ScienceDirect

Applied Geography

journal homepage: www.elsevier.com/locate/apgeog

Scale invariance of water stress and scarcity indicators: Facilitating cross-scale comparisons of water resources vulnerability

Shama Perveen [a,*], L. Allan James [b,1]

[a] School of Environmental and Public Affairs, University of Nevada Las Vegas, 4505 S. Maryland Parkway, Greenspun Hall #3119, Las Vegas, NV 89119, United States
[b] Department of Geography , University of South Carolina, 709 Bull Street, Callcott Building, Columbia, SC 29208, United States

Keywords:
Multi-scale analysis
Water resources vulnerability
Grid models
Falkenmark index
Criticality ratio

A B S T R A C T

Several indicators are commonly used to measure the degree of water resources vulnerability (e.g., water stress and scarcity) in different populations and regions. Little is known, however, about how these indicators respond to changes in the scale of data used to derive them. Two of the most widely used water resources vulnerability metrics, conventionally computed for mean annual values at the country level are *Falkenmark Index* (FI) for per capita water availability and the *Criticality Ratio* (CR) for water use to availability. This study computes FI and CR values at a wide range of scales and tests for trends with scale in three river basins: Missouri (North America), Danube (Europe) and Ganges (South Asia) Basins. Gridded sub-continental hydro-climatic data sets at 0.5° resolution are used and aggregated at multiple scales from 0.5° to 5.0°.

Analytical logic and empirical evidence show that mean grid-cell values of these vulnerability metrics are in fact scale-independent (scale-invariant) for a given basin. When unscaled variables like water availability and use are ratioed to variables that depend on area, such as population, their dependency on scale may be lost and they become spatially scaled variables. For example, grid-cell mean values of water availability are scale dependent, but grid-cell mean values of the ratio of water availability to population (i.e. FI) are not. This implies that, for a particular river basin, average water resources vulnerability computed by FI and CR at one scale should apply to all scales. This has tremendous implications to applied geographic studies of water resources, and is especially interesting since the unscaled variables used to derive the two indices are scale dependent and vary greatly with scale. The paper and findings highlight the multi-scale complexities of water resources and the geographic nature of water resources and vulnerability metrics.

Background

Vulnerability to environmental change and human events is a major determinant of global risk (Kasperson, 2001). The concept of vulnerability has been studied and applied in a wide range of disciplines (Tran, O'Neill, & Smith, 2010); often using different meanings and concepts of vulnerability, which has led to diverse methods of measuring it (Alwang, Siegel, & Jorgensen, 2001). Vulnerability of a region resulting from limited water resources availability and intensive water use, defined here as water resources vulnerability (WRV) has been a topic of several past studies. While WRV can be described through a number of attributes and indicators (Kulshreshtha, 1998), this study is concerned with assessments of WRV using two commonly used indicators – *Falkenmark Index* (FI) and *Criticality Ratio* (CR). The FI is defined as the average per capita water available per year (or social water stress), while the CR is computed as the ratio of mean water use to availability (technical water stress). In this study, the two indices are computed and tested for trends within three major river basins: the Missouri, Danube, and Ganges Basins, using a multi-scale approach.

In recent decades, against a growing alarm over 'water wars' (Shiva, 2002; Starr, 1992), several global agencies, national governments and NGOs have been concerned with emerging water 'crises' and potential water conflicts (FAO, 2003; UN, 2003). While most global-scale analyses and assessments of WRV have been cast at the country or regional scale (L'vovich & White, 1990; Wallace, 2000), several attempts have been made to analyze water resources at a more refined spatial scale with essentially the same indicators as those employed at the global or national scale (Alcamo, Döll, Kaspar, & Siebert, 1997; Meigh, McKenzie, Austin, Bradford, & Reynard, 1998; Vörösmarty, Green, Salisbury, &

* Corresponding author. Tel.: +1 803 629 3619.
E-mail address: sperveen@gmail.com (S. Perveen).
[1] Tel.: +1 803 777 6117.

0143-6228/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.apgeog.2010.07.003

Lammers, 2000). These studies illustrate the importance of using high spatial-resolution data to analyze local water scarcity problems. They highlight the fact that national-level analyses, by averaging over large areas, may greatly underestimate the number of people living under high (and low) water stress. For instance, Vörösmarty et al. (2000) showed that a 0.5° resolution analysis predicts 1.76 billion people under high water stress in 1995 compared to an estimate of only 0.45 billion indicated by a coarser national-scale analysis. The global-scale assessments, although a valuable starting point, present an averaged (lumped) view of the situation aggregated on a regional or country-wide basis. In practice, however, water shortages often become apparent first as occasional deficits at certain times of the year during periods of high demand or below-average rainfall and may only affect a narrow section of the population (Meigh, McKenzie, & Sene, 1999). Although several improvements have been made to national scale and annual average water scarcity analyses (Seckler, Amarasinghe, Molden, de Silva, & Barker, 1998), the applicability of these coarser scale data to local problems has insurmountable limitations. As high-resolution population and water resources data sets become increasingly available, a key research issue in vulnerability studies is the need to better grasp the complexities introduced by multi-scale analyses; i.e. how results of analysis at one scale can be used at another.

Why do multi-scale vulnerability assessments matter? Fundamentally, scale effects have long been studied in human geography as part of the modifiable areal unit problem or MAUP, which occurs in spatial analysis when area-based data are aggregated (Arbia, Benedetti, & Espa, 1996; Openshaw, 1984). If a statistic is calculated for two different sets of areal units that cover the same population, or sample, a difference will usually be observed even though the same basic data have been used in both analyses. One form of MAUP that has frequently been discussed is the so-called "ecological fallacy" (Robinson, 1950), which refers to inappropriate extrapolation of statistical relationships from aggregate to individual level. In experimental studies, it is rarely observed that the same sets of processes combine to affect variation at different scales (Legendre et al., 1997). It is more likely that different combinations of processes interact to affect patterns of heterogeneity at different scales, since the strength of the interaction between these variables will vary as spatial scale varies (Legendre et al., 1997). Thus, it may be more constructive to conduct studies that simultaneously examine the effect of processes across multiple spatial scales and then attempt to make interpretations of the relative importance of each across those scales. Several hydrologic studies have examined the effects of scale on means and variability. Wood (1998) describes a series of studies that demonstrated decreased variance in runoff, runoff ratio, and evapotranspiration with scale. Most such studies are based on changing the scale of model parameters and examining the effects on model output, so the effects of scale are integrated with a complex series of processes within the model. The study presented in this paper differs in that a single data set is resampled at varying scales and no other changes are made. Thus, it represents a controlled experiment in which the effect of scale is the sole factor governing the changes observed.

Two important implications therefore arise through multi-scale expressions of WRV. First, how results of analyses at one scale can be used for another is both fundamental to an understanding of processes and strategically important for water management decision-making in an appropriate and timely fashion (Tchiguirinskaia, Bonell, & Hubert, 2004). Secondly, knowledge of where water stress/scarcity occurs as a function of population and water availability and how it changes with scale may help with conflict resolution. Knowing and utilizing the scale at which water-related conflict is dominant may lead to a better understanding and consequently the development of appropriate management and policies (Victoria, Viegas Filho, Pereira, Teixeira, & Lanna, 2005). Practical planning and management of water resources requires models with greater spatial resolution that provide improved estimates of the number of people at risk and how these might change as the population grows (Wallace & Gregory, 2002).

This study is part of a much broader assessment of multi-scale variability and scaling trends using water resources and population data (Perveen & James, 2009). In that study, scaling functions were developed for certain variables like population, water availability, and use. The main aim of this paper is to apply those empirical functions to analyze means and variability in two commonly used WRV indicators used in this paper — *Falkenmark index* (FI) and *Criticality ratio* (CR). The findings presented here demonstrate that the grid-cell means of both metrics are scale-independent. Furthermore, this paper presents a theoretical proof that grid-cell means of these ratios are independent of scale. This has tremendous implications to applied geographic studies of water resources, and is especially interesting since the variables used to derive the two indices are scale-dependent. This study further provides an empirical and theoretical basis on which detailed study of the adapting capacity of the locale to WRV can be done.

Concepts and metrics of WRV

Humans are deeply integrated into the water cycle and their actions strongly determine the availability of water resources (Vörösmarty, 2002). As human influences on water supplies grow, societies will increasingly be placed in competition for scarce water resources and exposed to floods, droughts, pollution, public health problems, and economic stress. A proper understanding of anthropogenic impacts on the availability of water is central to integrated and sustainable water resources management.

A region becomes vulnerable to the availability of a natural resource if it cannot pursue its accepted policy goals at the desired level due to shortages (Kulshreshtha, 1993). The origins of this vulnerability may lie in various socio-economic or physical characteristics like population growth, economic situation, infrastructure, or climate variability. In most studies, measurement of WRV has been expressed as the annual availability of water per person (FI), percentage of available water withdrawn for use (e.g., CR), and the demand supply balance. If water stress or scarcity in a region is high, the population is more vulnerable to change. Explanations for vulnerability to water resources shortages can be complex and no single metric can completely express this vulnerability. For example, the simple vulnerability measures described in this paper do not include measures of resilience that express the ability of a society to recover from shortages, such as economic, educational, or demographic variables that could facilitate alternative supplies (e.g., virtual water). Yet, the indices examined here are conventional measures of WRV and the principles of multi-scale analysis explored here may be applicable to alternative metrics.

The indices conventionally employed to express WRV at the global scale are relatively simple to understand and robust in many respects (Rijsberman, 2006). The relation between total water availability and withdrawals has often been used to indicate the degree of mobilization in comparison to the total resource available (Falkenmark, Rockstrom, & Savenije, 2004). The higher the percentage of available water used by a country, the larger the costs in terms of necessary infrastructure. The experience in Europe has been that when water withdrawals reached 20 percent of the available water, the costs of infrastructure get high (Falkenmark & Lindh, 1976). Policy analysts have used the "Criticality Ratio" (CR) to express these technical and economic difficulties and recognize that when the CR exceeds a threshold of 0.4 a condition of severe

Please cite this article in press as: Perveen, S., James, L. A., Scale invariance of water stress and scarcity indicators: Facilitating cross-scale..., Applied Geography (2010), doi:10.1016/j.apgeog.2010.07.003

water stress ensues (Alcamo, Henrichs, & Rösch, 2000). The UN Comprehensive Assessment of the World's Freshwater Resources (UN/WMO/SEI, 1997) referred to the CR as *technical water stress* and introduced a scale to characterize its values (Table 1).

Demographers, on the other hand, have given special attention to the implications of population growth on socio-economic development. The *social* side of water stress is seen as critical in understanding implications of the different types of social disputes that may occur when a large number of people have to share a limited amount of available water. Thus, a *water crowding* scale was initially introduced to focus on the number of people sharing one flow unit of water, taken as one million cubic metres per year of renewable water (Falkenmark, 1986; FAO, 2000). To facilitate international comparisons, a set of intervals on this scale was introduced. For the upper limit, the most crowded water threshold was set at 2000 persons per flow unit and the term '*water barrier*' was introduced; understood as the maximum level that an advanced, irrigation-dependent country can sustain (Falkenmark et al., 2004). Later, engineers inverted the water crowding ratio to "available water per capita" so that thresholds were expressed as volumes of freshwater per capita. This index was widely adopted due to its simplicity and is now a standard measure of per capita water availability referred to as a *social water scarcity indicator* or "Falkenmark Index" (FI) (Table 2). This was renamed later by Population Action International as the *standard indicator* (Falkenmark & Widstrand, 1989).

Objectives and methods of the study

This paper is a component of a broader study on multi-scale assessments and spatial and statistical variability using water resources and population data. The variables chosen for this study is also used in the computation of WRV globally. Data, methods, and scaling functions developed for changes in water resources and population data with scale for the three basins, are described in detail in Perveen (2008), Perveen and James (2009). Statistical trend analysis of changes in mean values of variables with increasing scale (coarser grid-cell sizes) has demonstrated two classes of variables with distinctly different spatial behaviors – "unscaled" variables that tend to increase with scale (e.g., water availability, use) and "scaled" variables as ratios of an area-dependent factor (e.g., water availability per capita, population density) that decrease with scale. It is hypothesized that scaled and unscaled variables will behave differently due to differences in two conflicting theories: either variability increases due to increasing magnitudes of cell values or variability decreases due to spatial averaging within cells (Perveen, 2008) (Table 3).

The objective of this paper is to extend the research on effects of spatial heterogeneity and scale variability in water resources and population data on the metrics computed for WRV. In particular, the impacts of changing scale on two key WRV indices (FI and CR) will be demonstrated. Given that the FI and CR are ratios of two unscaled variables, it may seem intuitive they will change with scale. It will be shown, however, that the influence of scale on the two variables cancels out when they are expressed as a ratio. In the context of this study, *scale* is analogous to the 'spatial resolution' or 'grid-cell' size of the observations, and these terms will be considered to be synonymous. Similarly, 'fine' and 'coarse' will be used in conjunction with scale to represent the spatial extent of individual data elements; e.g., the size of grid cells or other structural elements.

There is no reason to assume that changes in variables with scale will be linear. Much research has cautioned against this assumption, which is particularly dangerous if scale affects different variables with varying magnitudes. The nature of scale change (linear, non-linear, etc.) can be critical to modeling or calculating water stress/scarcity (i.e. WRV) indicators, so specifying the behavior of variables at different scales is important (Perveen, 2008). This study identifies trends in variability for water resources variables and finds suitable scaling functions for the trends. Particularly useful is the identification of scaling trends for commonly used WRV indicators, which has not been attempted before. In the context of these research questions, several hypotheses are stated for empirical analysis of water resources and vulnerability indicators with scale (Table 3).

Hypothesis testing was conducted on datasets for the three river basins. The primary objective of the study was to empirically analyze the influence of scale on mean and variability metrics for water resources and population data, including the measures for WRV. The distinction between *unscaled* variables (population, water availability and water use) that are dependent on grid-cell size, and *scaled* variables (population density) that should be largely independent of cell size, is critical to this type of analysis because the two behave very differently. As discussed earlier, FI and CR are ratios of scale-dependent (unscaled) variables and might be expected (erroneously) to behave as scaled variables.

Statistical regression analysis using ordinary least squares (OLS) was conducted to test for scale effects on the variability of scaled and unscaled variables. Tests included various hypotheses that means and measures of statistical variability (standard deviation)

Table 1
Characterization of Criticality Ratio (technical water stress)

Percent withdrawal	Technical water stress
<10	Low water stress
10–20	Medium low water stress
20–40	Medium high water stress
>40	High water stress

Source: UN/WMO/SEI 1997

Table 2
Thresholds for Falkenmark Index (social water scarcity)

Original Falkenmark indicator for water crowding (Persons per flow unit[a]/yr)	Adapted 'water scarcity' index or 'Falkenmark indicator' (m^3/capita/yr)	Water stress implication
>600	<1700	Water stress
>1000	<1000	Chronic water scarcity
>2000	<500	Beyond the water barrier

[a] One flow unit = one million cubic metre.

Table 3
Statements of research hypotheses and their descriptions (italicized words indicate key variables being tested)

Hypothesis	Description
H1	Mean and variability (standard deviation) increases with scale for unscaled variables in all three basins
H1A	Mean increases with *scale* for *population*
H1B	Mean increases with *scale* for water *availability*
H1C	Mean increases with *scale* for *water use*
H1D	Standard deviation increases with *scale* for *population*
H1E	Standard deviation increases with *scale* for *water availability*
H1F	Standard deviation increases with *scale* for *water use*
H2	Mean and variability (standard deviation) decreases with scale for scaled variables in all three basins
H2A	Mean decreases with *scale* for *population density*
H2B	Mean decreases with *scale* for *water availability per capita*
H2C	Mean decreases with *scale* for *criticality ratio*
H2D	Standard deviation decreases with scale for *population density*
H2E	Standard deviation decreases with *scale* for *water availability per capita*
H2F	Standard deviation decreases with *scale* for *criticality ratio*

Please cite this article in press as: Perveen, S., James, L. A., Scale invariance of water stress and scarcity indicators: Facilitating cross-scale..., Applied Geography (2010), doi:10.1016/j.apgeog.2010.07.003

Table 4
Hypothesis testing and notation where scale is the *independent* variable and the *dependent* variables are: μ = mean; P = population; WA = water availability; WU = water use; PD = population density, WAC = water availability per capita and CR = criticality ratio. Also σ = standard deviation; ß = regression slope (e.g., $ß\mu_P$ is slope of regression line for mean (μ) of population density (PD) on scale).

Hypothesis	Notation: hypotheses (H_N); null hypotheses (H_o)
H1	Mean and variability (standard deviation) increases with scale for unscaled variables in all three basins
H1A	H_{1A}: $ß\mu_P > 0$; H_o: $ß\mu_P \leq 0$
H1B	H_{1B}: $ß\ \mu_{WA} > 0$; H_o: $ß\mu_{WA} \leq 0$
H1C	H_{1C}: $ß\ \mu_{WU} > 0$; H_o: $ß\mu_{WU} \leq 0$
H1D	H_{1D}: $ß_{\sigma P} > 0$; H_o: $ß_{\sigma P} \leq 0$
H1E	H_{1E}: $ß_{\sigma WA} > 0$; H_o: $ß_{\sigma WA} \leq 0$
H1F	H_{1F}: $ß_{\sigma WU} > 0$; H_o: $ß_{\sigma WU} \leq 0$
H2	Mean and variability (standard deviation) decreases with scale for scaled variables in all three basins
H2A	H_{2A}: $ß\mu_{PD} < 0$; H_o: $ß\mu_{PD} \geq 0$
H2B	H_{2B}: $ß\mu_{WAC} < 0$; H_o: $ß\mu_{WAC} \geq 0$
H2C	H_{2C}: $ß\mu_{CR} < 0$; H_o: $ß\mu_{CR} \geq 0$
H2D	H_{2D}: $ß_{\sigma PD} < 0$; H_o: $ß_{\sigma PD} \geq 0$
H2E	H_{2E}: $ß_{\sigma WAC} < 0$; H_o: $ß_{\sigma WAC} \geq 0$
H2F	H_{2F}: $ß_{\sigma CR} < 0$; H_o: $ß_{\sigma CR} \geq 0$

of variables change significantly with increasing grid-cell size (Table 4). In each hypothesis test, statistics from the OLS regression analysis were used to determine if the slope of the regression line (ß) expressing change in variability with resolution is significantly ($\alpha = 5\%$) different from zero. For both scaled and unscaled variables, the mean and variability statistics are dependent variables in univariate regressions on scale or grid-cell size, while scale is always the independent variable. Dependent variables were tested against null hypotheses, the notations for which are given in Table 4. The first series of null hypotheses (H1A–H1F) states that means and standard deviations of unscaled variables (population, water availability and water use) increase as scale becomes coarser. The second series of null hypotheses (H2A–H2F) states that means and standard deviations of scaled variables (population density, water availability per capita, and criticality ratio) decrease as scale coarsens.

Results

Mean and variability of *unscaled* variables were strongly and positively correlated with scale using both linear and power function models, although power functions were superior to linear models. For *scaled* variables, variability decreases at coarser scales, although the strengths of regressions were weaker and no single trend was identifiable.

Unscaled variables

The means and standard deviations (SD) of unscaled variables are positively correlated with scale as linear or power functions. The statistics for OLS regressions of linear function equations are shown in Table 5. With the significance level of the test set at $\alpha = 0.05$, the P values for the *t* statistics are significant for all unscaled variables under study for both mean and SD. It was hypothesized that mean and variability will increase with scale for unscaled variables (H1), so a one-tailed test was performed. Results of hypothesis tests for changes in mean and SD with scale for unscaled variables (i.e. the first series of null hypotheses, H1A–H1F, in Table 4) are summarized in Table 6. All P values of the *t* statistic are significant (≤0.05), so the null hypotheses that mean and SD decrease with scale is rejected in all cases for all three basins. Thus, the data from these controlled experiments with unscaled variables indicate that linear increasing functions of scale cannot be rejected on the basis of regressions.

Table 5
t statistics and *P* value for linear functions of mean and variability with scale for unscaled variables (where $\alpha = 0.05$)

Variables	Danube linear		Missouri linear		Ganges linear	
	t statistic	*P*	*t* statistic	*P*	*t* statistic	*P*
Mean water avail	25.60	.000	19.71	.000	20.23	.000
SD water avail	25.98	.000	14.61	.000	18.81	.000
Mean water use	25.60	.000	19.70	.000	20.23	.000
SD water use	14.73	.000	13.60	.000	16.21	.000
Mean population	25.60	.000	19.05	.000	20.23	.000
SD population	9.81	.000	22.60	.000	19.92	.000

Table 6
Summary table of the research hypotheses for mean and variability changes in unscaled variables (one-tailed test)

H1	Mean and Variability (SD) increases with scale for unscaled variables in all three basins	Accept/reject null hypothesis at $\alpha = 0.05$ (one-tailed test)
H1A	H_{1A}: $ß\mu_P > 0$; H_o: $ß\mu_P \leq 0$	Reject H_o in all 3 basins
H1B	H_{1B}: $ß\mu_{WA} > 0$; H_o: $ß\mu_{WA} \leq 0$	Reject H_o in all 3 basins
H1C	H_{1C}: $ß\mu_{WU} > 0$; H_o: $ß\mu_{WU} \leq 0$	Reject H_o in all 3 basins
H1D	H_{1D}: $ß_{\sigma P} > 0$; H_o: $ß_{\sigma P} \leq 0$	Reject H_o in all 3 basins
H1E	H_{1E}: $ß_{\sigma WA} > 0$; H_o: $ß_{\sigma WA} \leq 0$	Reject H_o in all 3 basins
H1F	H_{1F}: $ß_{\sigma WU} > 0$; H_o: $ß_{\sigma WU} \leq 0$	Reject H_o in all 3 basins

Mean values of unscaled variables were however seen to be best expressed as power functions of one-dimensional scale as shown for three river basins (Perveen & James, 2009; Table 7). While the linear equations are significant and the possibility that the data are linearly related to scale cannot be rejected on the basis of the regression analysis, both the visual fit of the lines and improved explained variance (R^2) (Perveen & James, 2009) indicate that power functions are a better model. Based on the use of power functions, it will be demonstrated theoretically in Section 4.3 that this scale-dependency of mean values of unscaled variables is lost when they are ratioed (in calculations of WRV indices). This has key implications to WRV studies.

Scaled variables

Most of the means and standard deviations (SD) of scaled variables are negatively correlated with scale but the relationships are weaker than with unscaled variables and the trends are inconsistent. The statistics for linear OLR regressions are shown in Table 8. Values of the *t* statistics for slopes of the linear equations derived for SD of scaled variables are negative. These SD correlations were significantly ($P < 0.05$) negative in slope, which signifies a decreasing trend in variability with scale. This corroborates the findings of Wood (1998) for scaled hydrologic variables of runoff (yield per unit area) and evapotranspiration (losses per unit area).

Table 7
Power functions for mean of unscaled variables (with scale) in the three river basins (X = scale in degrees)

River basin	Unscaled variables	Power functions (of means)
Missouri	Water availability (km^3)	$Y_{watavail} = 9.57\ X^{1.71}$
	Water use (km^3)	$Y_{watuse} = 0.22\ X^{1.71}$
	Population	$Y_{pop} = 59{,}629\ X^{1.71}$
Danube	Water availability (km^3)	$Y_{watavail} = 54.05\ X^{1.60}$
	Water use (km^3)	$Y_{watuse} = 0.42\ X^{1.60}$
	Population	$Y_{pop} = 669{,}357\ X^{1.60}$
Ganges	Water availability (km^3)	$Y_{watavail} = 71.40\ X^{1.66}$
	Water use (km^3)	$Y_{watuse} = 1.94\ X^{1.66}$
	Population	$Y_{pop} = 2{,}892{,}322\ X^{1.66}$

Source: Perveen and James 2009

Please cite this article in press as: Perveen, S., James, L. A., Scale invariance of water stress and scarcity indicators: Facilitating cross-scale..., Applied Geography (2010), doi:10.1016/j.apgeog.2010.07.003

Table 8
t statistic and *P* value for linear function of mean and variability with scale for scaled variables (where $\alpha = 0.05$)

Variables	Danube linear		Missouri linear		Ganges linear	
	t statistic	*P*	*t* statistic	*P*	*t* statistic	*P*
Mean pop density	3.582	.002	1.041	.159	−0.290	.389
SD pop density	−4.015	.001	−3.925	.001	−3.962	.001
Mean water avail per capita (m^3)	1.657	.062	−2.276	.021	−0.992	.171
SD water avail per capita (m^3)	−3.316	.003	−2.151	.027	−2.166	.026
Mean criticality ratio	−2.495	.014	−2.564	.013	−2.146	.027
SD criticality ratio	−1.946	.038	−2.187	.025	−1.931	.039

For the means, the values of the *t* statistic are mostly negative, except for the means of population density in the Missouri and Danube Basins and mean water availability per capita in the Danube basin. Using the *P* value for the *t* statistic, one-tailed tests of the hypothesis that variability decreases linearly with scale were conducted at a significance level of $\alpha = 0.05$. Based on the *P* values for regressions of the means of scaled variables on scale, the significance of these relationships was mixed (Table 9).

The null hypothesis was rejected for all three basins for mean values of criticality ratios and for SDs of population density, water availability per capita (or FI), and criticality ratio (Table 9). For the mean of population density and FI, the null hypothesis that these scaled variables are positively correlated with scale ($ß \geq 0$) could only be rejected for the Danube and Missouri Basins, respectively (Table 9). The null hypotheses could not be rejected for mean values of population density in the Missouri or Ganges or of FI in the Danube or Ganges; indicating a substantial possibility that the negative linear correlation model for means as a function of scale is inaccurate (H2 is incorrect) in these cases.

Unlike the unscaled variables, no one appropriate model could be found for fitting the changes in variability with scale within the range of observed values. While linear models of change in variability with scale are statistically significant, most of the plots appear to be curvilinear, especially for variability in FI and CR (Fig. 1a–f). Model fitting using SPSS shows that inverse functions of CR are the most appropriate model for changes in variability at multiple scales. For FI, however, no one best–fit model could be identified. Power functions seem the best for the Ganges basin, while inverse functions are more appropriate for the Danube and Missouri Basins. In short, the high signal-to-noise ratios in variability-scale relationships with scaled variables prevent identification of clear, consistent trends with scaled variables.

Table 9
Summary table of the research hypotheses for mean and variability changes in scaled variables (one-tailed test of linear model)

H2	Mean and variability (SD) decrease with scale for scaled variables in all three basins	Accept/reject null hyp at $\alpha = 0.05$ (one-tailed test)
H2A	H_{2A}: $ß\mu_{PD} < 0$; H_o: $ß\mu_{PD} \geq 0$	Danube – reject H_o Missouri – Fail to reject H_o Ganges – Fail to reject H_o
H2B	H_{2B}: $ß\mu_{WAC} < 0$; H_o: $ß\mu_{WAC} \geq 0$	Danube – Fail to reject H_o Missouri – reject H_o Ganges – Fail to reject H_o
H2C	H_{2C}: $ß\mu_{CR} < 0$; H_o: $ß\mu_{CR} \geq 0$	Danube – reject H_o Missouri – reject H_o Ganges – reject H_o
H2D	H_{2D}: $ß_{\sigma PD} < 0$; H_o: $ß_{\sigma PD} \geq 0$	Danube – reject H_o Missouri – reject H_o Ganges – reject H_o
H2E	H_{2E}: $ß_{\sigma WAC} < 0$; H_o: $ß_{\sigma WAC} \geq 0$	Danube – reject H_o Missouri – reject H_o Ganges – reject H_o
H2F	H_{2F}: $ß_{\sigma CR} < 0$; H_o: $ß_{\sigma CR} \geq 0$	Danube – reject H_o Missouri – reject H_o Ganges – reject H_o

Theoretical basis for scale independence of ratios of unscaled variables

The mean values of many spatially 'unscaled' variables increase as power functions of scale (grid-cell length); e.g., population, water use, and water availability (Table 7). Power functions of scale for these unscaled variables can facilitate computations of parameters commonly used to assess WRV at various scales. For instance, ratios of unscaled variables such as CR (water use/availability) and FI (water availability/population) can be expressed as a ratio of power functions:

$$R = a_1 S^{b1} / a_2 S^{b2} \tag{1}$$

where *R* is a ratio of scale-dependent variables, S is scale (or grid-cell size expressed as a one-dimensional variable such as distance or degrees; e.g., a 1-km grid or a 3° grid), and a_n and b_n are coefficients and exponents of the power functions, respectively. This ability to model means as scale-dependent power functions does not hold true, however, for variables scaled directly or indirectly to area (e.g., population density or water availability per capita) (Perveen & James, 2009).

It will now be shown that the exponents, b_1 and b_2, should be equal so that the scale components cancel out setting R equal to a constant. This indicates that such ratios of power functions are scale-independent. It was shown empirically that the exponents in power functions of scale computed for unscaled variables in this study are equal for a particular basin (Table 7; Perveen & James, 2009), and it can be shown mathematically that this should always be the case. Mean values of grid cells of different unscaled variables will increase with cell size at the same rate; i.e. as an inverse function of the number of cells. This is because the total of all cell values remains constant while the number of cells changes with scale:

$$Y_T = \sum Y_i \tag{2}$$

where Y_T is the total of a variable over the entire area (e.g., total water availability for a basin), and Y_i is the amount of the variable in grid cell *i*. The mean grid-cell value at a given scale is a function of the number of cells at that scale:

$$\overline{Y}_j = Y_T / N_j \tag{3}$$

where $\overline{Y}_j$ is the mean value of grid cells at scale *j*, and N_j is the number of grid cells at scale *j*. Since Y_T remains constant across all scales, the mean cell value for any variable is a simple inverse function of N_j. Thus, mean values of different variables will increase at the same rate as the number of cells decrease with aggregation from a fine to a coarse scale. This explains the constant exponent values for unscaled variables observed in Table 7.

If the exponents of power functions in the numerator and denominator of ratios of power functions of scale are equal, it follows that Equation (1) can be simplified to a constant by cancelling the scale values ($S^{b1}/S^{b2} = 1$):

$$R = a_1 / a_2 \tag{4}$$

where a_1/a_2 is a ratio of constants that reduces to a single value for the basin across all scales. This logic shows that ratios of power functions of scale that are developed from mean values of unscaled

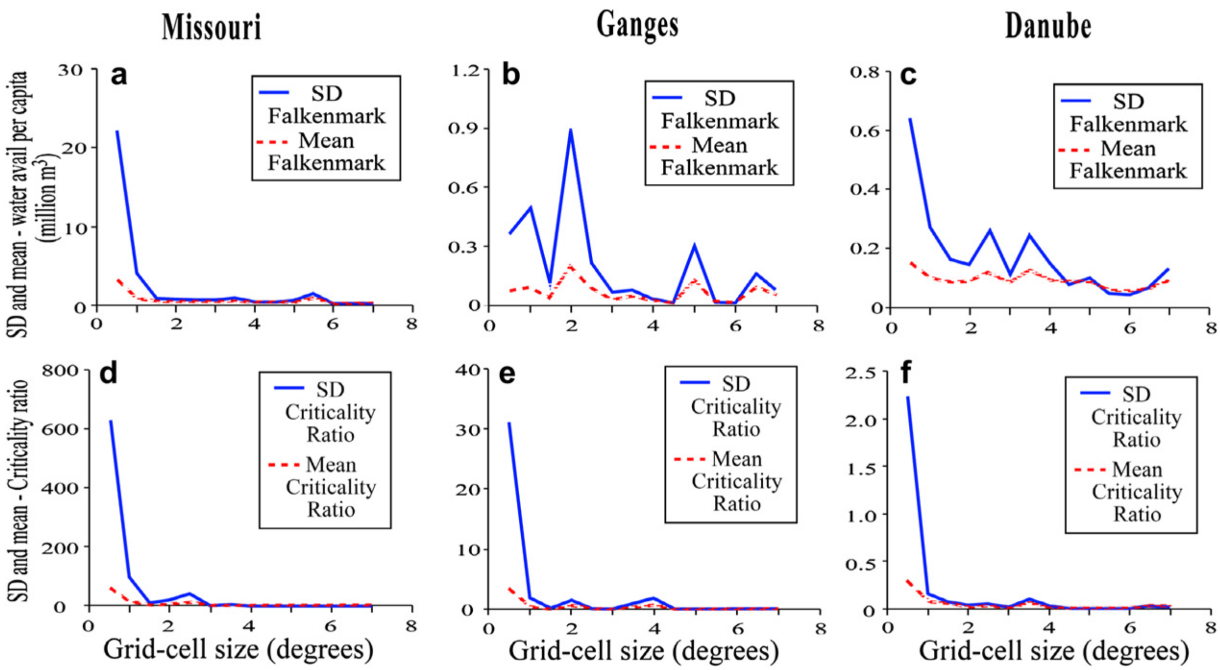

Fig. 1. (a–f). Scaled variables for the three river basins under study. Mean values (denoted by dotted red line) of water availability per capita (FI) in three basins (top row), and water use to availability (CR) (bottom row) do not display an overall trend with scale. Standard deviations (denoted by a solid blue line) of these variables may have a negative decreasing trend with scale, but this is strongly reliant on outliers at the fine, 0.5° scale. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

variables will be scale-independent. For example, mean water use and water availability for the Missouri Basin at any scale can be calculated using the power functions given in Table 7. Consequently, the mean CR for the basin can be computed as the ratio of those functions:

$$C_R = 0.22\ S^{1.71}/9.57\ S^{1.71} \tag{5}$$

where C_R is the CR at scale S (in degrees). The equation for the mean CR in the Missouri Basin simplifies to a constant indicating that it is independent of scale:

$$C_R = 0.22/9.57 = 0.0230 \tag{6}$$

This constant mean value of CR is valid for the MissouriBasin at any scale. It should also be valid regardless of the units of scale used to compute water use and availability as long as the scale units were the same for both parameters in Equation (1) and produced a statistically valid power function of scale. The same logic can be applied to the Danube and Ganges basins where the exponents are the same and cancel out so the CR is equal to the ratio of the coefficients of the water use and water availability functions. A similar relationship is also true for the FI (per capita water availability, in m^3) that can be calculated for any scale as the ratio of the coefficients for the water availability and population power functions with scale. It follows, thus, that the mean CR and FI for a given basin are scale-independent ratios (Table 10). This is noteworthy, since the FI is not a dimensionless number like the CR.

Table 10
Values of Criticality Ratio and the Falkenmark indicator for the three basins

River basin	Criticality Ratio (CR)	Falkenmark Index (FI) (m^3/capita)
Missouri	0.0230	160,492.38
Danube	0.00777	80,749.14
Ganges	0.0272	24,686.05

Testing the scale independence of mean ratios

Mean values of FI and CR, plotted against scale for the three river basins under study, indicate mixed results (Fig. 1a–f). In contrast, variability, computed as standard deviation (SD), shows systematic trends with scale. The statistical results present in Tables 8 and 9 shows that linear regressions of mean FI for the Danube and Ganges Basins were not significant but were significant for the Missouri Basin. All three linear regressions of mean CR values with scale had significantly negative trends. All of the plots with the exception of mean FI in the Ganges Basin are strongly influenced by the point associated with the finest scale (0.5°). The graphs in Fig. 1 clearly illustrate this effect on the negative trend in mean values with scale. Apparently, the original values of these data sets contain isolated large values that are averaged out by the first pass of data aggregation but drive up mean and SD values at the fine scale. Whether or not such outliers (speckling) represent real phenomenon or whether they are common in other data sets, goes beyond the scope of this paper. When the first (0.5°) values are eliminated from the mean FI and CR data, regression slopes are less negative and are/are not significant. Thus, the results of the FI and CR tests of power function ratios are not fully conclusive. Clearly the presence of outliers drives up variance at the finest scale.

Understanding the fundamental differences between different types of data and different basins is essential for water resources modelers and scientists dealing with multi-scale analyses. Scaled vs. unscaled variables obviously behave quite differently, and behaviors with scale vary greatly between basins. For instance, in the Ganges Basin, variability in water availability per capita (FI) changes rapidly with scale. This may be because, of the three basins chosen for study, the Ganges has the highest population density at 360 people km^{-2}. Also, the basin is dotted with highly populated cities and urban areas, which may lead to random and unsystematic changes in variability as data is aggregated from fine to coarse scales. A daunting challenge before the environmental and resources communities will be to address long-standing problems

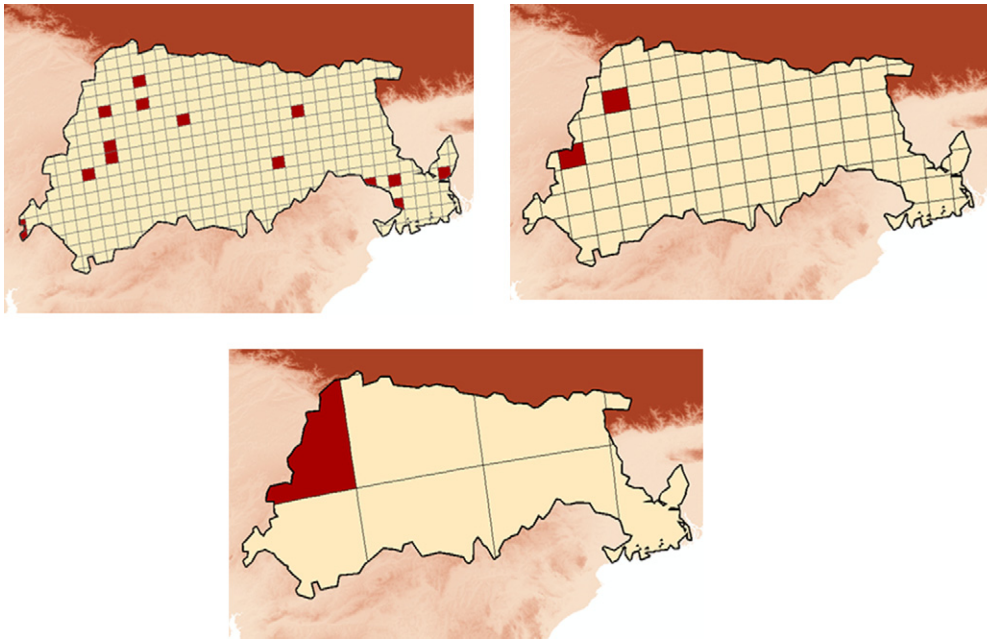

Fig. 2. Grid cells having water stress in the Ganges basin clockwise from upper left at 0.5°, 1° and 5°, respectively. Though the population under water stress at each scale is almost the same, the spatial pattern is different at each.

of heterogeneity in water resources through the use of multi-scale spatial and quantitative techniques.

Future directions of this research

New gridded water resources and related datasets are being developed that are rich in detail. Understanding the effects of changing scale will enable the retention of the detail of these fine-scale datasets while allowing up-scaling to regional and continental scales for broader applications. Patterns of water stress and scarcity vary greatly with scale. At coarse scales (large grid-cell sizes), some areas where WRV is prevalent are lost and other areas emerge. For a basin like the Ganges (Fig. 2), this disparity in information with scale is quite misleading in water resources studies, since this basin is dotted with highly populated cities, especially along the Ganges River. Water availability, use, and population in the basin have been shown to be strongly dependent on scale (Perveen & James, 2009).

Multi-scale analyses inform policy makers of the complexities of water resources data and the geographic nature of water resources and vulnerability metrics, such as water stress and water scarcity. To achieve a comprehensive picture of the vulnerability of freshwater resources to global change, however, several major data and technical challenges must be confronted directly. Bio-geophysical data associated with Earth systems models are beginning to provide an important new source of information for global water resources studies. These data are available at a variety of spatial and temporal scales, but have not yet been well integrated with the data sets used in traditional water resources assessments. In addition to information on the physical geography of water availability and water quality, socio-economic data sets are critically needed for comprehensive water resources assessments. These data needs include spatially distributed statistics on urban and rural demographics, daily and monthly water use by sector (agricultural, industrial, or municipal), water-use efficiencies, economic indicators, investment in infrastructure, irrigable lands, water quality, degree of water and wastewater treatment, and human health statistics. Some fine-scale socio-economic datasets have nonetheless been made available for GDP, flood mortality risk, drought risk, and population projection.

Linking physical water scarcity assessment models with relevant social, economic, institutional and legal schemes that affect water use and management is a major challenge. Physical water shortages are closely linked with human well-being and poverty, and are being connected operationally with social and environmental factors (e.g., via the Water Poverty Index of Sullivan, 2000, 2002). Fine-scale gridded data sets are needed and are being developed to allow merging of physical and social databases. The challenge is to integrate a multi-scale framework into the physical and social dimensions of global-change research that enables data to be extracted and analyzed at a wide variety of scales and results to be compared across scales with an understanding of the effects of scale. In light of the findings presented here, multi-scale analyses involve systematic but complex statistical behavior that must be understood to facilitate the multi-scale integration of climate change, hydrologic, water resources, and socio-economic data.

Discussion and conclusions

Various exploratory and statistical tests demonstrated the effects of scale – as expressed by grid-cell size – on water resources variables and metrics used to calculate WRV. The key findings can be summarized as the recognition of simple systematic behaviors of mean values and variability of certain types of variables with scale. These behaviors may provide guidance towards broader fundamental spatial relationships governing multi-scale analyses in water resources research and other geographically oriented studies.

Power functions of scale for mean grid-cell values of unscaled variables in a region can be derived that explicitly define the increase of these variables with scale (grid-cell size). This indicates that the behavior of certain variables in multi-scale analyses can be predicted directly as a result of simple mathematical properties of spatial scaling. It also explains oft-noted dependencies of mean and variability on scale. The power functions have equivalent exponents because the increase is geometrically determined by grid-cell size. When power functions are ratioed, therefore, the variables raised to

Please cite this article in press as: Perveen, S., James, L. A., Scale invariance of water stress and scarcity indicators: Facilitating cross-scale..., Applied Geography (2010), doi:10.1016/j.apgeog.2010.07.003

equivalent exponents cancel out, so mean values of FI and CR, key WRV indices, should theoretically be scale-invariant for a given region or basin. This theory of scale invariance in mean values of certain ratios was partially demonstrated through empirical testing, except that mean values at the finest grid-cell size often formed a large outlier. The scale invariance theory was valid through the range of scales tested except at the finest scale (0.5°). More analysis and testing of this phenomenon are needed. These scale-invariant relationships for mean values do not hold true for other metrics such as variance or standard deviation because the exponents vary; viz., Equations (2) and (3) do not apply.

Understanding the spatial behavior of phenomena in a multi-scale analysis will be of increasing importance as new data and technologies arise. The motivation to address questions of spatial variation in water availability and demands at multiple scales has grown in the face of increasing resource pressures. Ultimately, water resources scientists should be able to use empirically derived indicators of spatial means and variability to build geographically specific models and evaluate uncertainties associated with water stress and scarcity calculations at a given scale. When mean values are scale-dependent, such as with water availability, difficulties and uncertainties may arise in interpretations and comparisons between studies conducted at different scales unless the scale-dependencies can be quantified. For instance, mean values of water availability computed at a national level from 1 km^2 grid cells cannot be compared with mean values computed at a local scale from 1 ha grid cells, unless adjustments are made for scale. Moreover, the differences in such comparisons become more pronounced in regions of high heterogeneity. This study provides a quantitative means of adjusting for the effects of scale that can be applied retroactively or on the fly in models. Just as importantly, this study reveals two classes of variables that behave very differently with regards to scale. In contrast with unscaled variables that increase with grid-cell size, mean values of scaled variables do not show the same positive scale independency. The magnitudes of change introduced by scaling are less with these variables, and the trends with scale tend to be less consistent but opposite in sign (negative).

Scale-based analysis is timely and the spatial and statistical analyses of water resources and socio-economic data will help to place existing ideas about water management in an accurate geographic perspective with a greater appreciation for the importance of scale. This study has obvious applied implications concerning the ability to detect and map social consequences of impending water shortages and food crises resulting from water scarcity. With such grave implications of a global nature, studies of freshwater availability and use at different scales become imperative. With integrated water resources and river basin management paradigms emerging in water resources, water governance at a variety of scales is growing in importance.

References

Alcamo, J., Döll, P., Kaspar, F., & Siebert, S. (June 1997). *Global change and global scenarios of water use and availability: An application of WaterGAP1.0*. Germany: Center for Environmental Systems Research (CESR), University of Kassel.

Alcamo, J., Henrichs, T., & Rösch, T. (2000). *World water in 2025 – Global modeling and scenario analysis for the world commission on water for the 21st century*. Kassel World Water Series 2. Germany: Center for Environmental Systems Research, University of Kassel.

Alwang, J., Siegel, P. B., & Jorgensen, S. L. (2001). *Vulnerability: A view from different disciplines*. World Bank. SP Discussion Paper No. 0115.

Arbia, G., Benedetti, R., & Espa, G. (1996). Effects of the MAUP on image classification. *Geographical Systems, 3*, 123–141.

Falkenmark, M. (1986). Fresh water – time for a modified approach. *Ambio, 15*, 192–200.

Falkenmark, M., & Lindh, G. (1976). *Water for a starving world*. Boulder, CO: Westview Press.

Falkenmark, M., Rockstrom, J., & Savenije, H. (2004). *Balancing water for human and nature: The new approach in ecohydrology*. UK: Earthscan. 247 pp.

Falkenmark, M., & Widstrand, C. (1989). Population and water resources: a delicate balance. *Population Bulletin, 47*(3), 1–35.

FAO. (2000). *New dimensions in water security – Water, society and ecosystem services in the 21st century*. Rome: Land and Water Development Division, Food and Agriculture Organization of the United Nations. FAO Report: AGL/MISC/25/2000. 82 pp.

FAO. (12 March 2003). *No global water crisis – But many developing countries will face water scarcity*. Rome: FAO News Stories.

Kasperson, R. (2001). Vulnerability and global environmental change. *Newsletter of the International Human Dimensions Programme on Global Environmental Change, 2*. IHDP Update.

Kulshreshtha, S. N. (1993). *World water resources and regional vulnerability: Impact of future changes*. IIASA. Research Report RR-93–010, June. 137 pp.

Kulshreshtha, S. N. (1998). A global outlook for water resources to the year 2025. *Water Resources Management, 12*, 167–184.

Legendre, P., Thrush, S. F., Cummings, V. J., Dayton, P. K., Grant, J., Hewitt, J. E., et al. (1997). Spatial structure of bivalves in a sandflat: scale and generating processes. *Journal of Experimental Marine Biology and Ecology, 216*(1), 99–128.

L'vovich, M. I., White, G. F., Belyaev, A. V., Kindler, J., Koronkevic, N. I., Lee, T. R., et al. (1990). In B. L. Turner (Ed.), *The Earth as transformed by human action* (pp. 235–252). Cambridge: Cambridge Univ. Press.

Meigh, J. R., McKenzie, A. A., Austin, B. N., Bradford, R. B., & Reynard, N. S. (1998). *Assessment of global water resources – Phase II. Estimates of present and future water availability in Eastern and Southern Africa*. Wallingford, UK: Center for Ecology and Hydrology.

Meigh, J. R., McKenzie, A. A., & Sene, K. J. (1999). A grid-based approach to water scarcity estimates for Eastern and Southern Africa. *Water Resources Management, 13*, 85–115.

Openshaw, S. (1984). *The modifiable areal unit problem*. Norwich: Geo Books.

Perveen, S. (2008). *Multi-scale effects on spatial metrics in global water resources data: implications to water stress*, Dissertation submitted in fulfilment of Ph.D. Columbia: Department of Geography, University of South Carolina.

Perveen, S., & James, L. A. (2009). Multi-scale effects on spatial variability metrics in global water resources data. *Water Resources Management*. doi:10.1007/s11269-009-9530-2.

Rijsberman, F. R. (2006). Water scarcity: fact or fiction? *Agricultural Water Management, 80*, 5–22.

Robinson, W. S. (1950). Ecological correlations and the behavior of individuals. *American Sociological Review, 15*, 351–357.

Seckler, D., Amarasinghe, U., Molden, R., de Silva, R., & Barker, R. (1998). *World water demand and supply, 1990 to 2025: Scenarios and issues*. International Water Management Institute. Research Report 19. 40 pp.

Shiva, V. (2002). *Water wars: Privatization, pollution and profit*. Cambridge, MA: South End Press.

Starr, J. R. (1992). Water wars. *Foreign Policy, 82*, 17–36.

Sullivan, C. A. (2000). Redefining the water poverty index – a commentary. *Water International, 25*(4).

Sullivan, C. A. (2002). Calculating a water poverty index. *World Development, 30*, 1195–1210.

Tchiguirinskaia, I., Bonell, M., & Hubert, P. (Eds.). (2004). *Scales in hydrology and water management*. International Association of Hydrological Sciences (IAHS). No 287, 170 pp.

Tran, L. T., O'Neill, R. V., & Smith, E. R. (2010). Spatial pattern of environmental vulnerability in the mid-Atlantic region. *Applied Geography, 30*, 191–202.

UN. (2003). *Water for people, water for life – UN World Water Development Report (WWDR)*. UNESCO/Berghahn Books/UN.

UN/WMO/SEI. (1997). *Comprehensive assessment of the freshwater resources of the world*. Report prepared for the 5th Session of the UN Commission on Sustainable Development. Document E/CN. 17/1997/9. United Nations. Stockholm: UN/WMO/Stockholm Environment Institute.

Victoria, F. B., Viegas Filho, J. S., Pereira, L. S., Teixeira, J. L., & Lanna, A. E. (2005). Multi-scale modeling for water resources planning and management in rural basins. *Agricultural Water Management, 77*, 4–20.

Vörösmarty, C. J. (2002). Global water assessment and potential contributions from earth system science. *Aquatic Sciences, 64*, 328–351.

Vörösmarty, C. J., Green, P., Salisbury, J., & Lammers, R. B. (2000). Global water resources: vulnerability from climate change and population growth. *Science, 289*, 284–288.

Wallace, J. S. (2000). Increasing agricultural water use efficiency to meet future food production. *Agriculture, Ecosystems and Environment, 82*, 105–119.

Wallace, J. S., & Gregory, P. J. (2002). Water resources and their use in food production systems. *Aquatic Sciences, 64*(2002), 363–375.

Wood, E. F. (1998). Scale analyses for land surface hydrology. In G. Sposito (Ed.), *Scale dependence and scale invariance in hydrology* (pp. 1–29). Cambridge: Cambridge University Press.

Please cite this article in press as: Perveen, S., James, L. A., Scale invariance of water stress and scarcity indicators: Facilitating cross-scale..., Applied Geography (2010), doi:10.1016/j.apgeog.2010.07.003